Temperature measurement by emissive power sensing

Principle, implementation and … pitfalls

Jean-Claude Krapez

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return on innovation

Outline

- Basics : Planck’s law, Wien’s law …
- Emissivity-Temperature Separation problem (ETS)
  - Pyrometry
    - single-color, bispectral pyrometry
    - multispectral pyrometry
  - ETS in airborne/satellite remote sensing
    - atmosphere compensation
    - spectral smoothness method
    - multi-temperature method
  - Bayesian perspective
- Conclusion

Foreword

“Success Is Going from Failure to Failure Without Losing Your Enthusiasm” *(statement erroneously attributed to W. Churchill - http://quoteinvestigator.com/2014/06/28/success/)*

This presentation reviews a series of methods aimed at providing a measurement of surface temperature through EM radiation sensing. A substantial number of them prove to be ineffective or, better said, show unpredictable success/failure, depending on the emissivity spectrum of the sensed material.

Listing only successful achievements would amount to sending a message as: “All problems have been solved, don’t bother, you just have to implement those solutions!”

For the present topic, this cannot be the case.

Spending some time for analyzing former failures is important for three reasons:
1- the poor performance of a given method is not always manifest at the onset;
2- if this enabled you to avoid making the same mistakes and to save time!

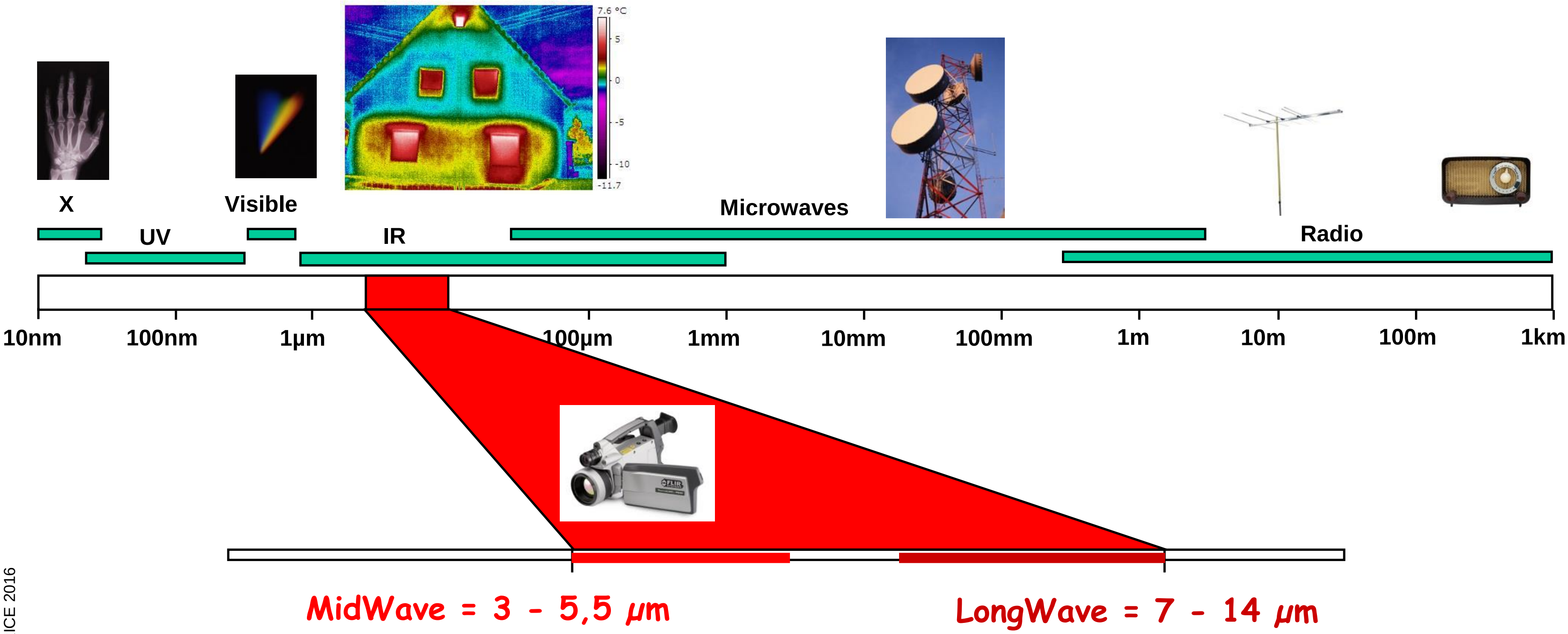
3- it shows that research is still needed on this subject!…

Thermal radiation

Matter emits EM radiation

Intensity increases with **temperature**

→ Monitoring of emitted radiation offers a mean for **temperature measurement**



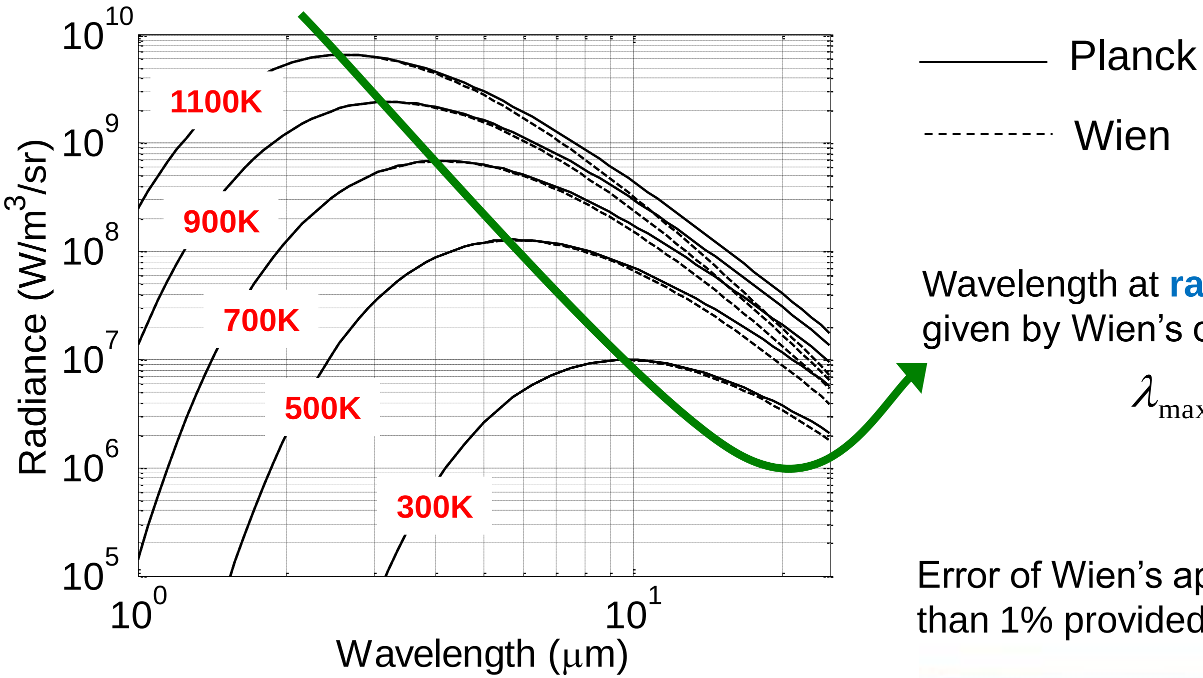
Thermal radiation sensing

Advantages of the radiation sensing method :

- **non-contact**
- **surface** to **sub-surface** probing (opaque or semi-transparent material)
- **rapid** : detectors with up to GHz bandwidth (and even higher)
- **long distance** measurement (airborne and satellite remote sensing, astronomy)
- **point detectors** (local measurement or 2D images by mechanical scanning) to **focal plane arrays** (instantaneous 2D images)
- **spectral** measurement also allows materials discrimination

Basics (1/4)

- **Blackbody**: perfect absorber, perfect emitter
- Spectral radiance given by **Planck's law**: $$B(\lambda,T)=\frac{C_1}{\lambda^5}\frac{1}{\exp\left(\frac{C_2}{\lambda T}\right)-1}$$
- **Wien's** approximation: $$B_W(\lambda,T)=\frac{C_1}{\lambda^5}\exp\left(-\frac{C_2}{\lambda T}\right)$$



Wavelength at **radiance maximum** given by Wien's displacement law:

$$\lambda_{\max}T = 2898\ \mu mK$$

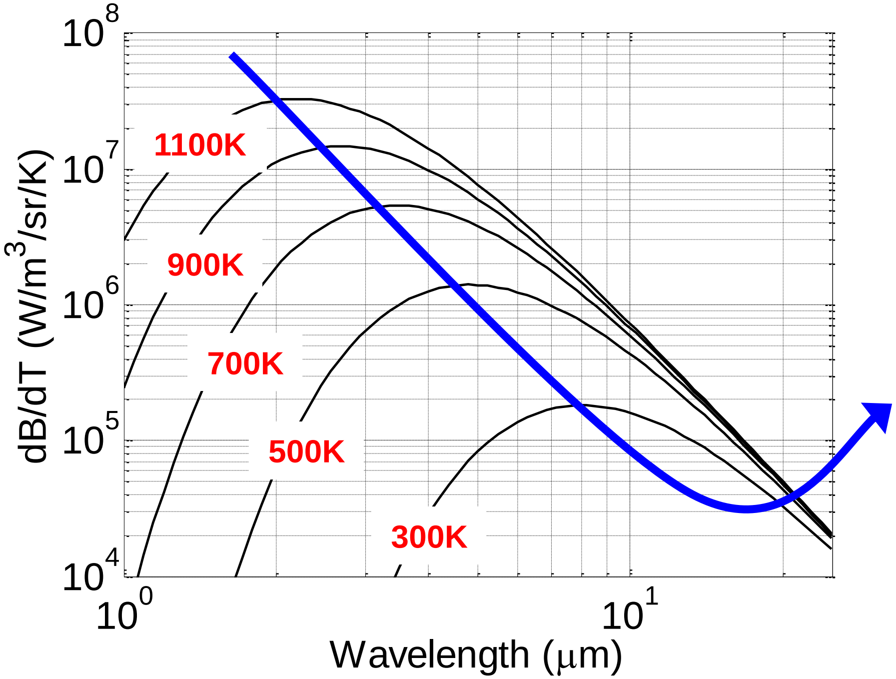
Error of Wien's approximation is less than 1% provided that $\lambda T < 3124\ \mu mK$

Wavelength selection for temperature measurement

- Maximum of radiance given by Wien's displacement law: $\lambda_{\max}T = 2898\ \mu mK$

$$\lambda_{\max} = \begin{cases} 9.7\mu m & at \quad T = 300K \\ 2.9\mu m & at \quad T = 1000K \end{cases}$$

- Radiance sensitivity to temperature (***absolute* sensitivity**): $\dfrac{\partial B}{\partial T}$



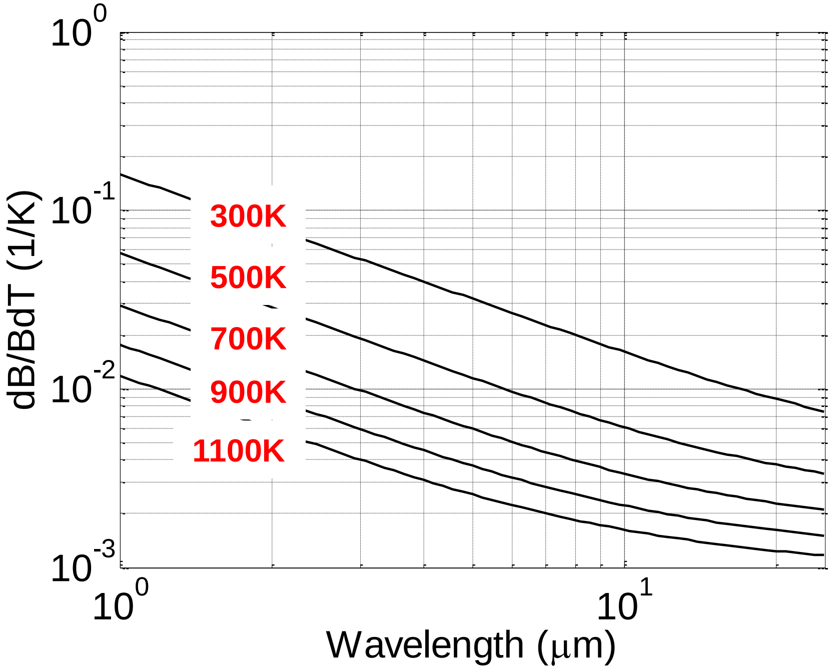
Maximum corresponds to: $\lambda_{\max}T = 2410\ \mu mK$

$$\lambda_{\max} = \begin{cases} 8.0\mu m & at \quad T = 300K \\ 2.4\mu m & at \quad T = 1000K \end{cases}$$

Basics (3/4)

Wavelength selection for temperature measurement

- Radiance sensitivity to temperature (***relative* sensitivity**): $\frac{1}{B}\frac{\partial B}{\partial T}$



At T = 300K :
2%/K radiance increase at 8μm
16%/K radiance increase at 1μm

Advantage of performing measurements at **short wavelengths** (sensitivity is nearly in **inverse proportion** to wavelength)

Interest in **visible** pyrometry or even in **UV** pyrometry ?

Basics (4/4)

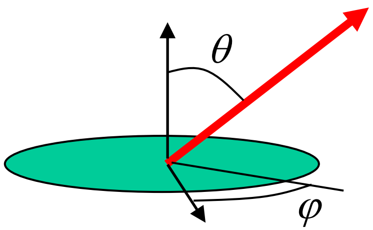
Real materials (**non-perfect emitters**)

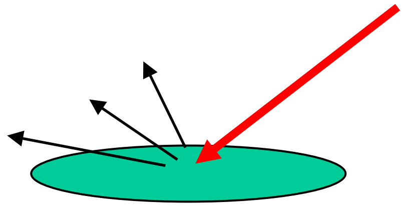
- with respect to **blackbody**, the emitted radiance $L(\lambda, T, \theta, \varphi)$ is reduced by a factor called **emissivity**:

$$L(\lambda, T, \theta, \varphi) = \varepsilon(\lambda, T, \theta, \varphi) B(\lambda, T) \qquad 0 \le \varepsilon \le 1$$

- emissivity depends on wavelength, temperature, and direction
- second Kichhoff's law between **emissivity** and **absorptance**:

$$\varepsilon(\lambda, \theta, \varphi) = \alpha(\lambda, \theta, \varphi)$$





- relation between **absorptance** and **directional hemispherical reflectance** from the **energy conservation law** for an opaque material (the energy that is not absorbed by the surface is reflected in all directions):

$$\alpha(\lambda, \theta, \varphi) + \rho^{'\cap}(\lambda, \theta, \varphi) = 1$$

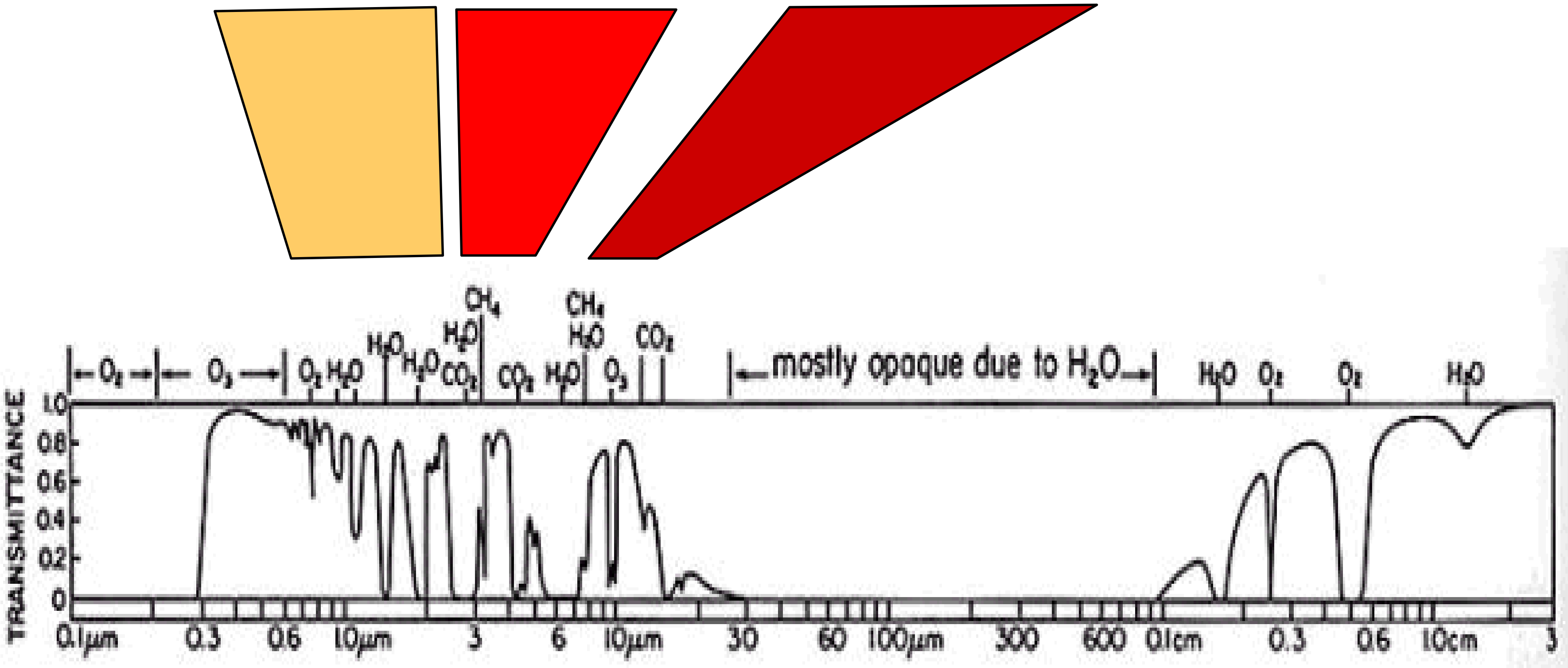
Emissivity can be inferred from a **reflectance measurement** (integrating sphere)
Drawback : need to bring the integrating sphere close to the surface

Radiation sensing is dependant on the **atmosphere transmission**, i.e. on the absorption bands of air constituents : H_2O, CO_2, O_3, CH_4, …

MidWave: 3 - 5,5 µm

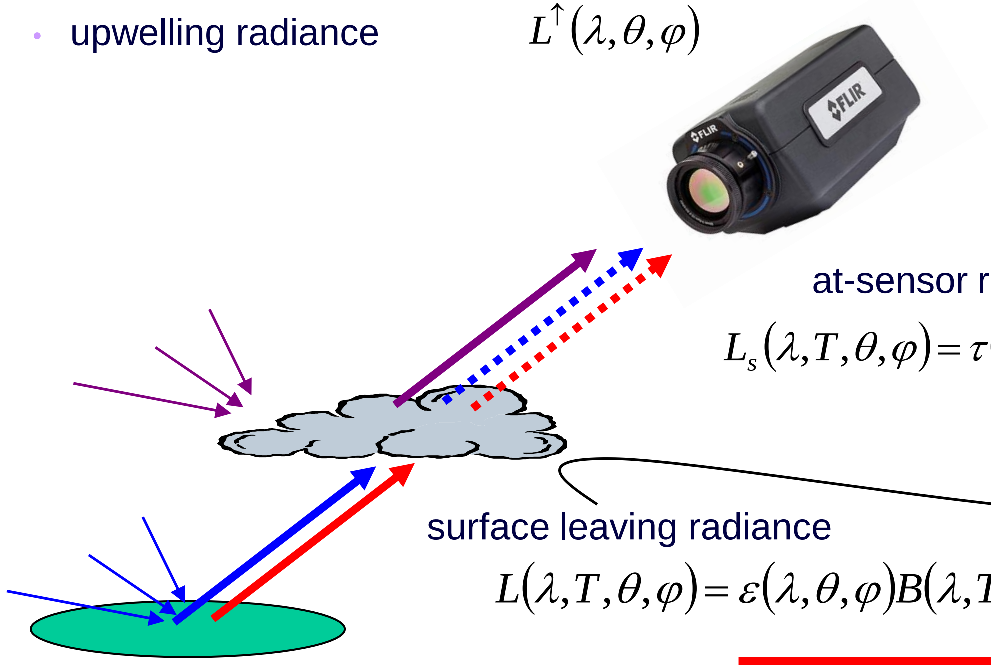
ShortWave: 0.7 - 2,5 µm

LongWave: 7 - 14 µm



Contributors to the optical signal

- the surface reflects the incoming radiation (non-perfect absorber)
  - downwelling radiance: $L^{\downarrow}(\lambda,\theta_i,\varphi_i)$
  - bidirectional reflectance : $\rho''(\lambda,\theta,\varphi,\theta_i,\varphi_i)$
- the radiation leaving the surface is attenuated along the optic path (absorption, scattering by atmosphere constituents: gases, aerosols – dust, water/ice particles)
  - transmission coefficient : $\tau(\lambda,\theta,\varphi)$
- atmosphere emits and scatters radiation towards the sensor
  - upwelling radiance $L^{\uparrow}(\lambda,\theta,\varphi)$



at-sensor radiance

$$L_s(\lambda,T,\theta,\varphi)=\tau(\lambda,\theta,\varphi)L(\lambda,T,\theta,\varphi)+L^{\uparrow}(\lambda,\theta,\varphi)$$

surface leaving radiance

$$L(\lambda,T,\theta,\varphi)=\varepsilon(\lambda,\theta,\varphi)B(\lambda,T)+\int_{2\pi}\rho''(\lambda,\theta,\varphi,\theta_i,\varphi_i)L^{\downarrow}(\lambda,\theta_i,\varphi_i)\cos\theta_i d\Omega_i$$

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First considered case

- Pyrometry of high temperature surfaces
  - sensor at close range (limited or even negligible atmosphere contributions)
  - environment much colder than the analyzed surface

$$L_s(\lambda,T,\theta,\varphi)=\underline{\varepsilon(\lambda,\theta,\varphi)}B(\lambda,T)$$

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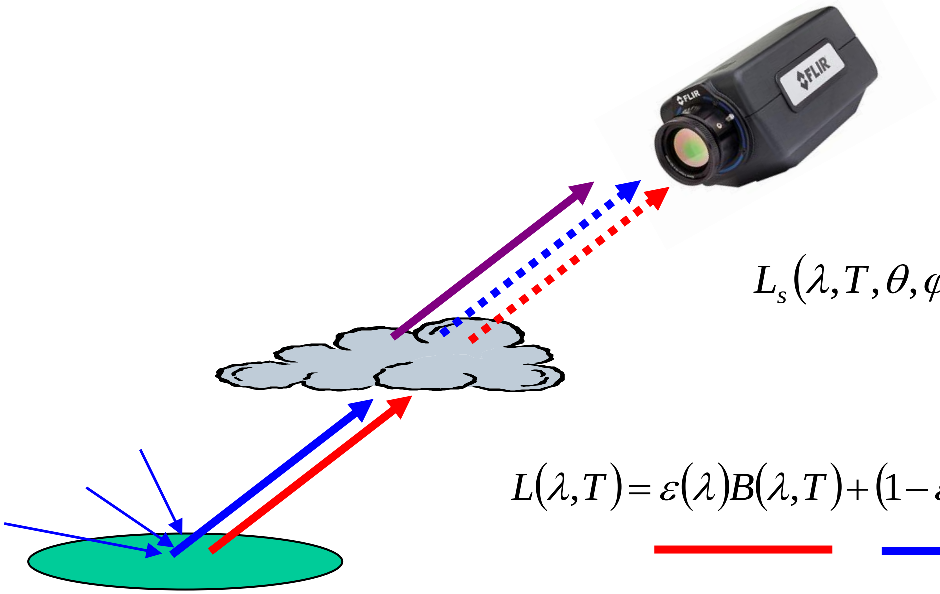
Second considered case

- Airborne/satellite remote sensing
  - hypothesis of lambertian surface: isotropic reflectance $\Longrightarrow$ isotropic emissivity
    - mean downwelling radiance $$L^{\downarrow}(\lambda) = \frac{1}{\pi} \int_{2\pi} L_{env}(\lambda, \theta_i, \varphi_i) \cos\theta_i \, d\Omega_i$$
  - need for **atmosphere compensation** step



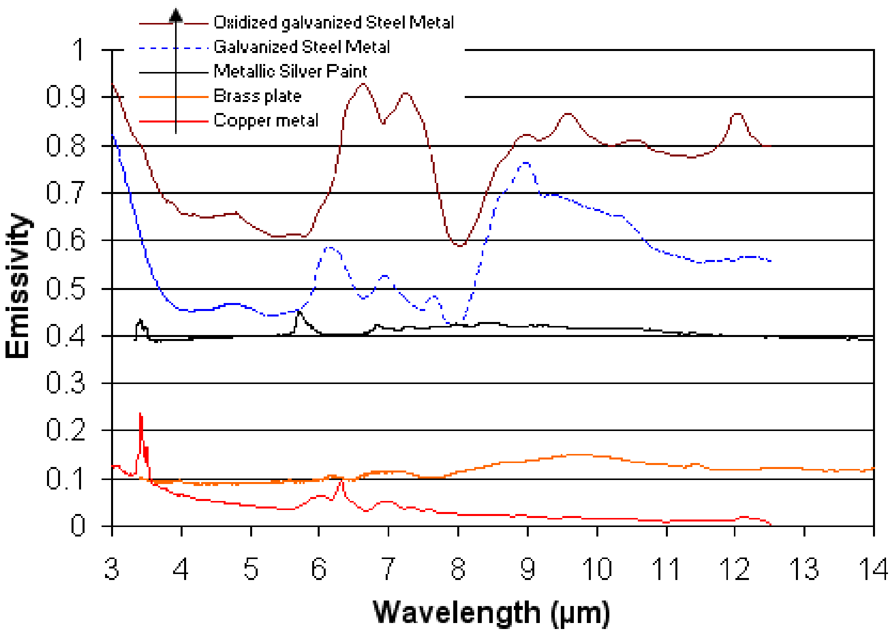
$$L_s(\lambda, T, \theta, \varphi) = \tau(\lambda, \theta, \varphi) L(\lambda, T) + L^{\uparrow}(\lambda, \theta, \varphi)$$

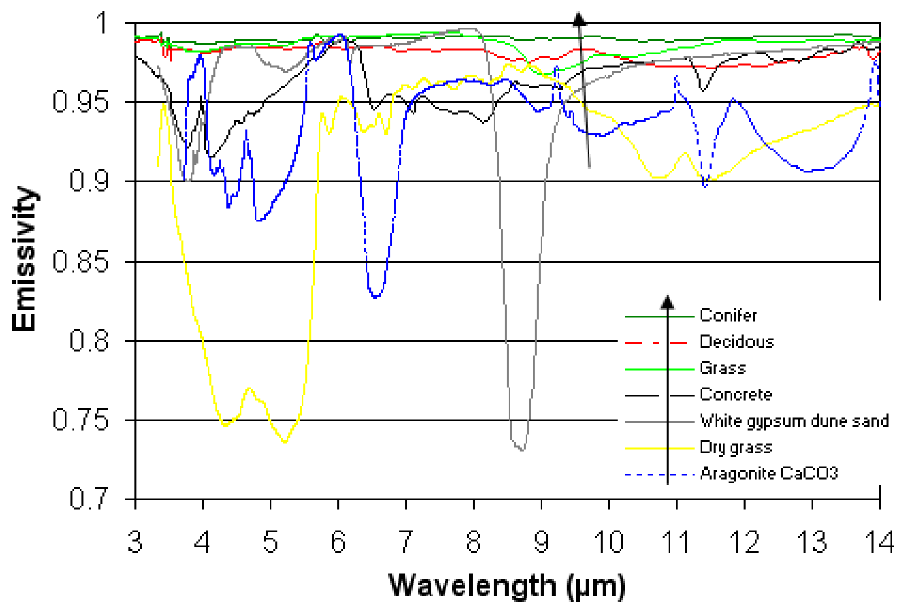
$$L(\lambda, T) = \varepsilon(\lambda) B(\lambda, T) + (1 - \varepsilon(\lambda)) L^{\downarrow}(\lambda)$$

What about emissivity ?

In all cases we need an information on emissivity to get temperature

- relations for emissivity : **only for ideal materials**, for example Drude law for pure metals ($\varepsilon(\lambda) \propto \lambda^{-1/2}$ satisfactory only for $\lambda > 2\mu m$, not valid for corroded or rough surfaces)
- databases for specific materials **in particular state** of roughness, corrosion, coatings, contaminant, moisture content …





- **The only practical solution : <u>simultaneous</u> temperature and emissivity evaluation**

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Single-color pyrometry

- Measurement is performed in a narrow or large spectral band
- In both cases, after sensor calibration, the retrived radiance is of the form

$$L_s(\lambda, T) = \varepsilon(\lambda) B(\lambda, T)$$

One equation, two unknown parameters

One has to estimate the emissivity (**a priori knowledge**)

- **Sensitivity** of temperature to an **error in emissivity estimation**:

$$\frac{dT}{T} = -\left(\frac{T}{B}\frac{dB}{dT}\right)^{-1}\frac{d\varepsilon}{\varepsilon} \approx -\frac{\lambda T}{C_2}\frac{d\varepsilon}{\varepsilon}$$

at 1µm and T= 1100K : -0.8K/% error
at 10 µm and T= 300K : -0.6K/% error

- **Sensitivity to emissivity error drops at shorter wavelengths** ⇒ advantage in working at in the visible or UV spectrum
- However, the signal drops too !

⇒ a compromise is needed

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Two-color pyrometry (1/4)

- **Adding a new wavelength**:
  - adds an **equation**
  - adds an **unknown parameter,** namely the emissivity a this additional wavelength
- Two spectral signals: $\begin{cases} L(\lambda_1,T)=\varepsilon(\lambda_1)B(\lambda_1,T) \\ L(\lambda_2,T)=\varepsilon(\lambda_2)B(\lambda_2,T) \end{cases}$
- by ratioing the signals: $\ln\left(L_1\lambda_1^{\ 5}\right)-\ln\left(L_2\lambda_2^{\ 5}\right)=\ln\left(\dfrac{\varepsilon_1}{\varepsilon_2}\right)-\dfrac{C_2}{\lambda_{12}T}$

effective wavelength: $\lambda_{12}=\dfrac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}$

- The problem **can be solved** if one has a knowledge about the **emissivity ratio** (less restrictive than the common « **greybody** » assumption : $\varepsilon(\lambda_1)=\varepsilon(\lambda_2)$)
- **Sensitivity** of temperature to an **error in emissivity estimation**:

Single color

$$\frac{dT}{T}\approx-\frac{\lambda T}{C_2}\frac{d\varepsilon}{\varepsilon}$$

Bi-color

$$\frac{dT}{T}\approx-\frac{\lambda_{12}T}{C_2}\left(\frac{d\varepsilon_1}{\varepsilon_1}-\frac{d\varepsilon_2}{\varepsilon_2}\right)$$

The effective wavelength is higher than λ_1,λ_2 : **bad news** !!

2- and 3- color pyrometry vs 1-color pyrometry

	1-color	2-color	3-color
Input	ε_1	$\dfrac{\varepsilon_2}{\varepsilon_1}$	$\dfrac{\varepsilon_2{}^2}{\varepsilon_1\varepsilon_3}$
Input (log(ε))	value	« slope »	« curvature »
Error amplification on temperature A	$-\dfrac{T}{C_2}\lambda_1$	$-\dfrac{T}{C_2}\dfrac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}$	$-\dfrac{T}{C_2}\times\dfrac{\lambda_1\lambda_2\lambda_3}{\lambda_3(\lambda_2-\lambda_1)-\lambda_1(\lambda_3-\lambda_2)}$
Example 1 λ_i	8µm	8µm; 9µm	8µm; 9µm; 10µm
Example 1 $A(K/\%)$	**-0,5**	**-4,5**	**-22,5**
Example 2 λ_i	9µm	9µm; 10µm	9µm; 10µm; 11µm
Example 2 $A(K/\%)$	**-0,56**	**-5,6**	**-30,9**

T=300K

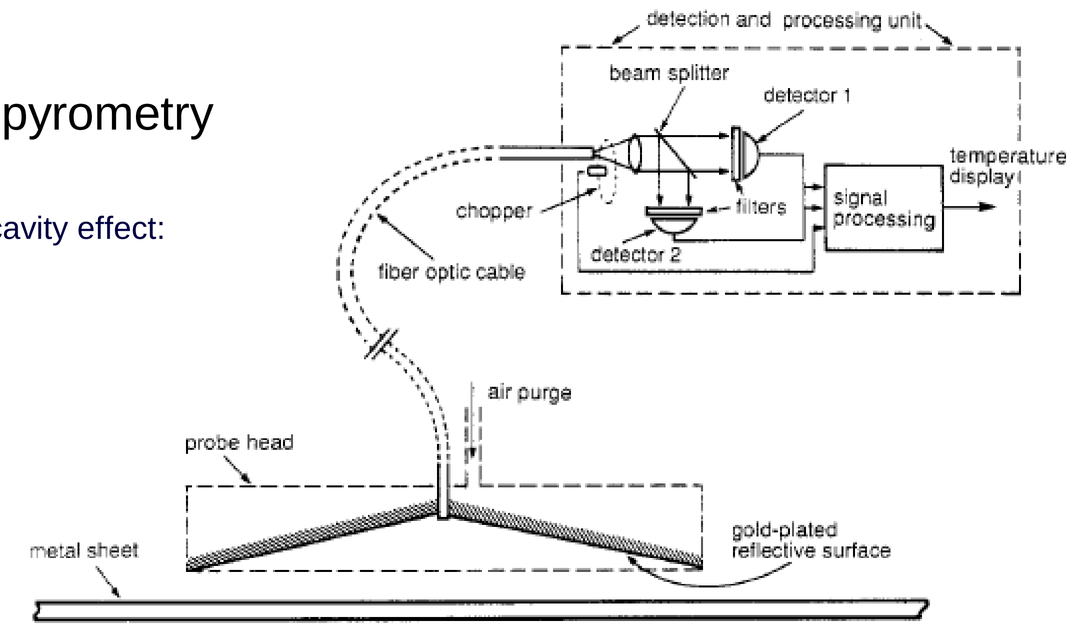
Two-color pyrometry (2/4)

- Sensitivity to emissivity errors can be reduced by **spreading** the two wavelengths
  ⇨ (false) "dilemma" between spreading the wavelengths and the classical "greybody" assumption
- Advantage of ratio pyrometry over single color pyrometry : **immunity to partial occultation, to variations of optical path transmission**

- Emissivity-enhanced 2-color pyrometry

Additional reflective surface for introducing a cavity effect:

- **increase of both apparent emissivities**
- **reduction of spurious reflections**



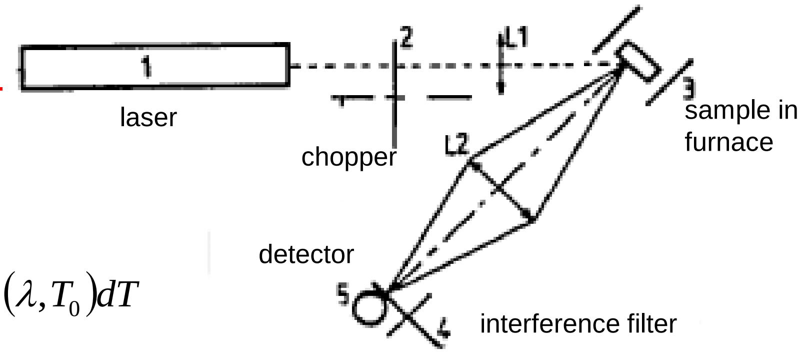
J.-C. Krapez at al., 1990

Two-color pyrometry (3/4)

- 2-color photothermal pyrometry:

  A laser is used for periodically heating the surface.
  A lock-in detection is implemented to capture
  the modulated radiance **deprived from any reflection**.

$$L(\lambda,T)=\varepsilon(\lambda)B(\lambda,T)+(1-\varepsilon(\lambda))L^{\downarrow}(\lambda)$$



1st order development: $B(\lambda,T_0+dT)=B(\lambda,T_0)+\frac{\delta B}{\delta T}(\lambda,T_0)dT$

The signal (amplitude) ratio is: $\frac{\varepsilon(\lambda_1)\partial B/\partial T\,(\lambda_1,T_0)}{\varepsilon(\lambda_2)\partial B/\partial T\,(\lambda_2,T_0)}$

T. Loarer at al., 1990
S. Amiel , 2014

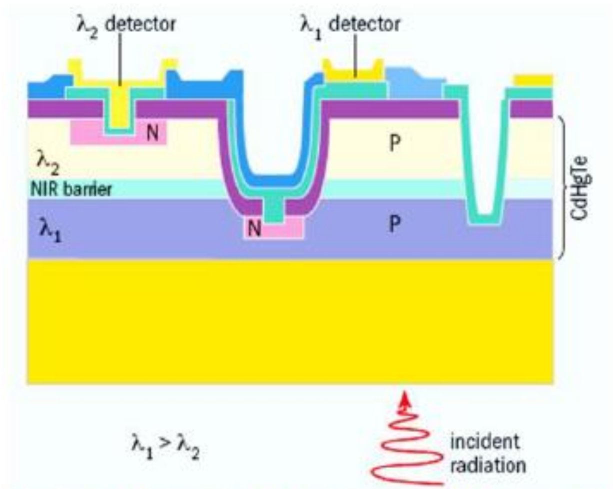


Figure IV-1. Architecture de la détection bicolore [Villermet 2012].

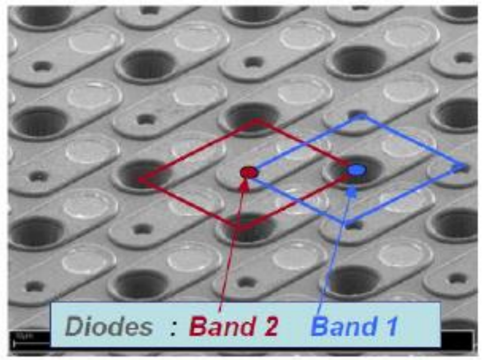


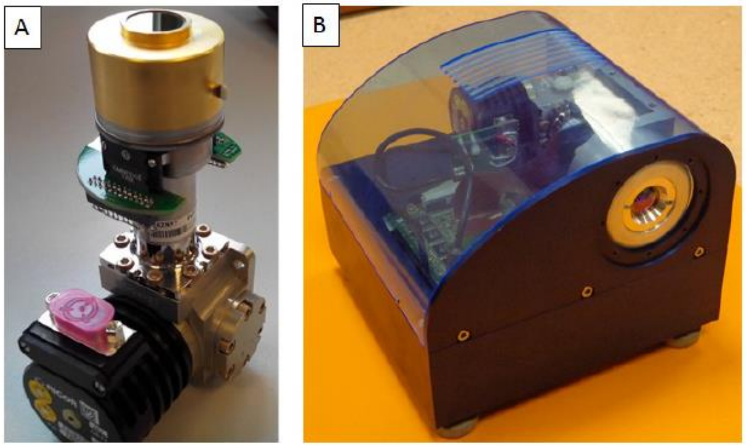
Figure IV-2. Illustration du décalage des deux matrices de pixels.

Bispectral MWIR camera by SOFRADIR :
Band 1 : 3.5-4.0µm
Band 2 : 4.6-5.0µm
640x505 pixels
Staggered sensitive areas
24µm pitch

S. Amiel , 2014

Two-color pyrometry (4/4)

- **MWIR + LWIR   two-color camera**
  Prototype developed by OSMOSIS
  (joint laboratory SOFRADIR+ONERA)
  640x512 pixels
  Staggered sensitive areas
  24 µm pitch



Band 1 : 3.5-5 µm
NETD: 30 mK

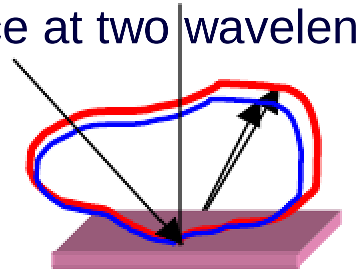
Band 2 : 7-9,5 µm
NETD: 39 mK

G. Druart et al., 2014

Two-color pyroreflectometry

D. Hernandez et al., 2005, 2009, 2014

Indirect evaluation of emissivity via directional reflectance at two wavelengths



$$L(\lambda,T)=\varepsilon^{\perp}(\lambda)B(\lambda,T)$$

$$\varepsilon^{\perp}(\lambda)=1-\rho^{\perp\cap}(\lambda)$$ directional hemispherical reflectance

$$\rho^{\perp\cap}(\lambda)=\pi\eta(\lambda)\rho^{\perp\perp}(\lambda)$$ **diffusion factor** : $\eta(\lambda)\approx\eta$ is assumed **independent of wavelength**

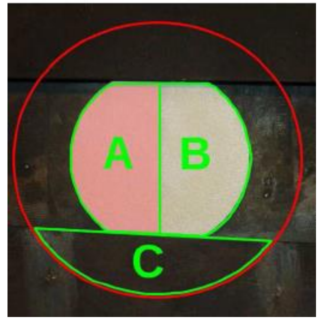


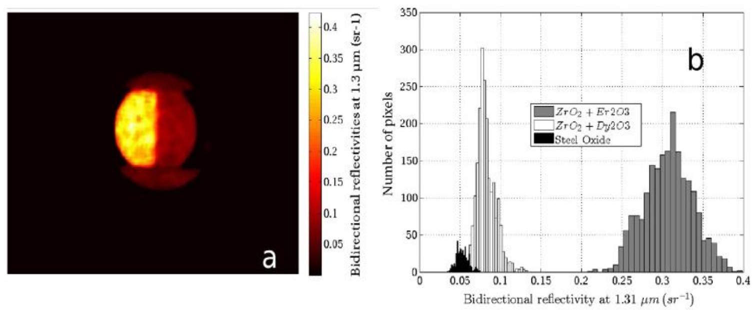
Fig. 1. Photograph of the complex scene mounted on the heating element. Item A: Erbium oxide part ($ZrO_2 + Er_2O_3$). Item B: Dysprosium oxide part ($ZrO_2 + Dy_2O_3$). Item C: Steel Oxide flange.

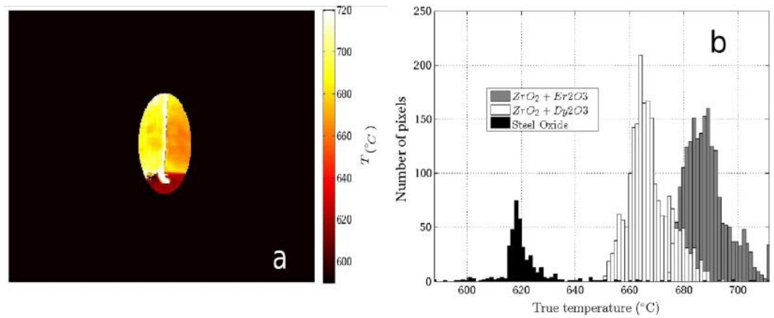
Two radiance measurements:

$$L(\lambda_i,T)=\left(1-\pi\eta\rho^{\perp\perp}(\lambda_i)\right)B(\lambda_i,T)\quad i=1,2$$

\+ **two measurements of dir-dir reflectance:** $\rho^{\perp\perp}(\lambda_i)\quad i=1,2$ with 20mW laser diodes at 1.31 and 1.55µm

The two left unknowns: η and temperature T are determined from the two radiance measurements





Multiwavelength pyrometry (MWP)

- Emissivity-temperature **separation** is essentially an **underdetermined inverse problem:**

N observables

$$L_s(\lambda_i, T) = \varepsilon(\lambda_i) B(\lambda_i, T) \qquad i = 1, N$$

N unknown parameters

1 unknown parameter

  whatever the number of wavelengths, there is always one **more** unknown parameter than available equations

- Two types of methods:
  - reduce **by one** the number of degrees of freedom of the discretized emissivity spectrum
    - N equations, N unknowns → the problem should be solvable (at first glance…)

      → **interpolation-based method**
  - regularization by using a much **lower-order emissivity model** (continuous or step functions)
    - N equations, M unknowns (M<<N)

      → **least-squares based method**

- Persistent **controversy**: does **MWP** really bring an advantage with respect to single-color or two-color pyrometry ?

Multiwavelength pyrometry. Interpolation-based method (1/3)

"**Just as needed**" regularization : the N emissivity values $\ln(\varepsilon_i)$ are represented by only N-1 parameters, e.g. the N-1 coefficients of a N-2 degree polynomial

By considering the Wien approximation and taking the logarithm, Coates showed that this may lead to "**catastrophic**" results:

$$\ln\left[L(\lambda_i, T)\lambda_i^5 / C_1\right] = \ln(\varepsilon_i) - C_2/\lambda_i T \qquad i = 1, N \qquad (1)$$

Emissivity model

induced temperature bias

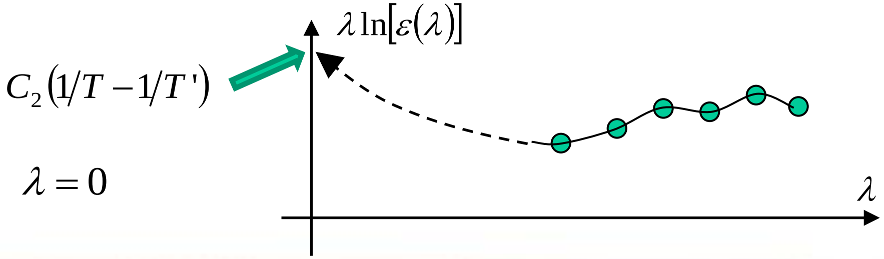
$$\ln\left[L(\lambda_i, T)\lambda_i^5 / C_1\right] = \sum_{j=0}^{N-2} a_j \lambda_i^{\,j} - C_2/\lambda_i T' \qquad i = 1, N \qquad (2)$$

$$(1)\times\lambda_i - (2)\times\lambda_i$$

$$\lambda_i \ln(\varepsilon_i) = \sum_{j=0}^{N-2} a_j \lambda_i^{\,j+1} + C_2\left(\frac{1}{T} - \frac{1}{T'}\right) \qquad i = 1, N$$

The constant parameter of the polynomial of degree N-1 passing through the N values $\lambda_i \ln(\varepsilon_i)$ gives the "temperature error" $C_2(1/T - 1/T')$

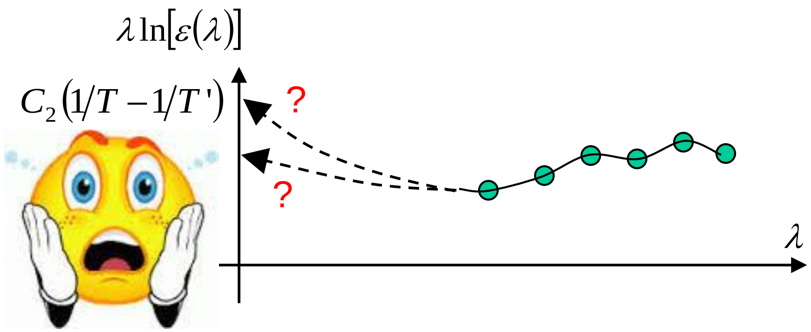
Real (unknown) emissivity values



i.e. **extrapolation result at** $\lambda = 0$

Multiwavelength pyrometry. Interpolation-based method (2/3)

There would be ***no error*** if a N-2 degree polynomial could be found passing ***exactly*** through the N values $\ln(\varepsilon_i)$ → **highly unlikely** !



Therefore, in general, one is exposed to the **deleterious** properties of **polynomial extrapolation**. Unfortunately, extrapolation based on polynomial interpolation leads to **increasingly high errors as the polynomial degree rises** !

→ **unpredictably high errors when adding new wavelengths**

Previous errors are **systematic**, i.e. **method errors** (they are observed even with an errorless signal !).

Same bad results are observed in the presence of **measurement errors** (they actually add to the previous ones !).

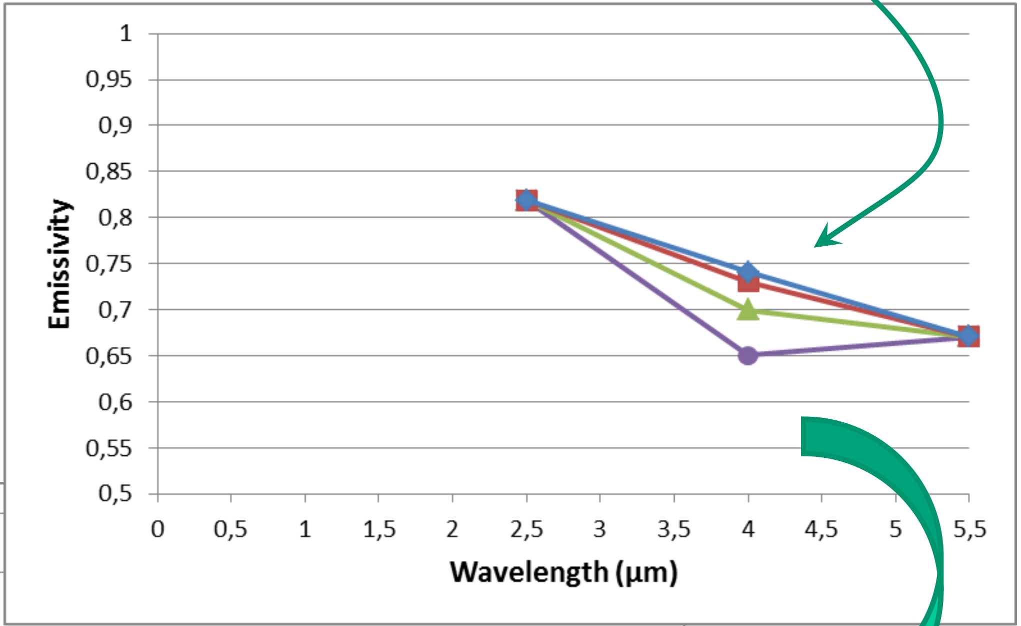
→ The calculated temperatures are **increasingly sensitive to measurement errors** as the **number of channels increases** : (kind of) **OVERFITTING** problem

Multiwavelength pyrometry. Interpolation-based method (3/3)

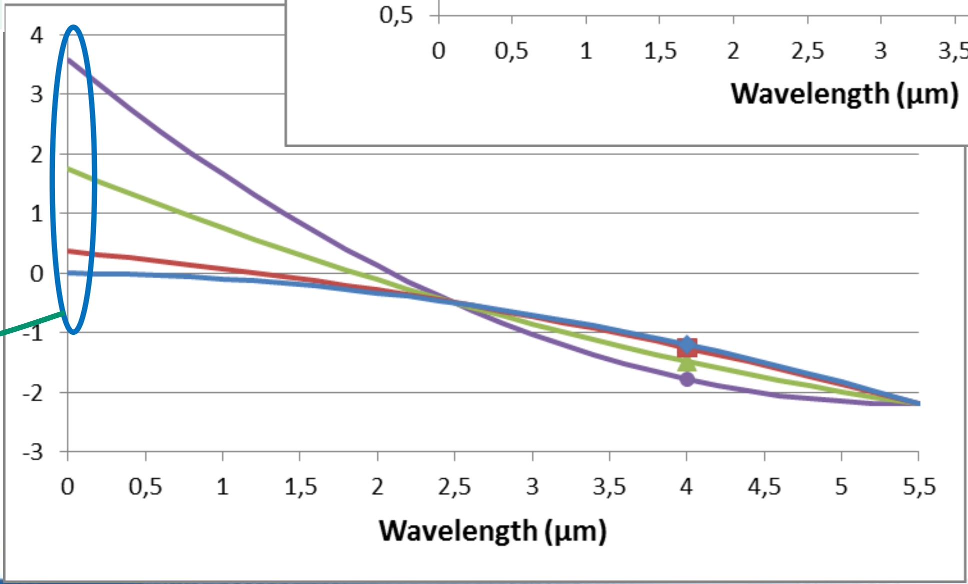
Example of **tricolor** pyrometry when $\ln(\varepsilon_i)$ progressively **departs from linearity**

Error in temperature (K)

$\Delta\varepsilon_2$	T=600K	T=1000K
0	0	0
-0,01	9	26
-0,05	47	137
-0,1	105	331



$\lambda \ln[\varepsilon(\lambda)]$



$C_2(1/T - 1/T')$

Multiwavelength pyrometry. Low-order emissivity models (1/2)

A (*seamingly* adequate) remedy: **reduce the model complexity** !

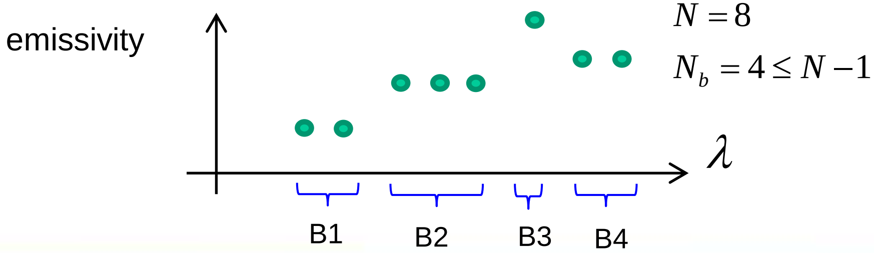
Some examples of **lower order** models :

$$\varepsilon(\lambda_i) = \sum_{j=0}^{m} a_j \lambda_i^{\,j} \quad i = 1,\ldots,N \quad m < N-2$$

$$\ln[\varepsilon(\lambda_i)] = \sum_{j=0}^{m} a_j \lambda_i^{\,j} \quad i = 1,\ldots,N \quad m < N-2$$

$$\varepsilon(\lambda_i) = 1/\left(1 + a_0 \lambda_i^{\,2}\right) \quad i = 1,\ldots,N$$

- Polynomials of $\lambda^{1/2}$ or $\lambda^{-1/2}$ for $\ln[\varepsilon(\lambda)]$
- Functions involving the brightness temperature $T_R = B^{-1}(\lambda, L_s)$
- Sinusoïdal function of wavelength
- Step function (**grey-band model** with N_b bands).
    - 2 or more channels per grey band
    - limiting case : $N_b = N-1$ with N-2 single-channel bands and 1 **dual channel** band



Multiwavelength pyrometry. Low-order emissivity models (2/2)

measurement error (noise)

Observable : $Y_i = \ln\left[L(\lambda_i, T)\lambda_i^5 / C_1\right] + e_i$

Wien approximation

Polynomial approx. of $\ln[\varepsilon(\lambda_i)]$

Minimizing the weighted sum $\sum_{i=1}^{N} \sigma_i^{-2}\left(Y_i - \left(\sum_{j=0}^{m} a_j \lambda_i^{\,j} - \frac{C_2}{\lambda_i T}\right)\right)^2$

→ Linear least squares problem

measurement error (noise)

Observable : $Y_i = L(\lambda_i, T) + e_i$

Planck's law

Polynomial approx. of $\varepsilon(\lambda_i)$

Minimizing the weighted sum $\sum_{i=1}^{N} \sigma_i^{-2}\left(Y_i - B(\lambda_i, T)\sum_{j=0}^{m} a_j \lambda_i^{\,j}\right)^2$

→ Non-linear least squares problem

Multiwavelength pyrometry. Linear least squares problem (1/5)

One is looking for the polynomial coefficients and the temperature such that:

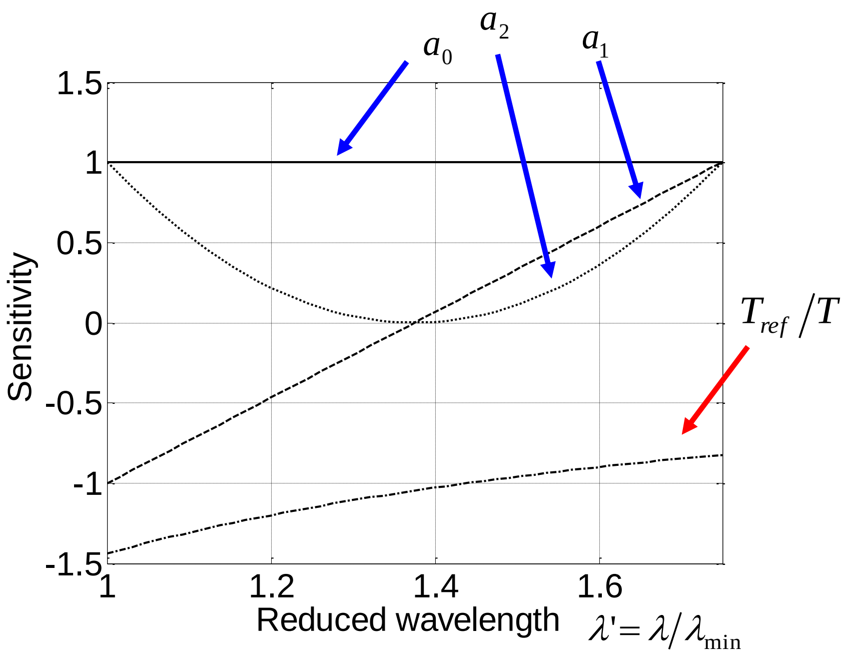
$$\hat{\mathbf{P}} = \begin{bmatrix} \hat{a}_0 & \dots & \hat{a}_m & \hat{T} \end{bmatrix}^T = \arg \underset{a_j,T}{Min} \sum_{i=1}^{N} \left(Y_i - \left(\sum_{j=0}^{m} a_j \lambda_i^{\ j} - \frac{C_2}{\lambda_i T} \right) \right)^2$$

Parameter reduction for numerical purposes:

$$\lambda_i^* = 2\frac{\lambda_i - \lambda_{\min}}{\lambda_{\max} - \lambda_{\min}} - 1 \qquad P_T^* = T_{ref}/T \quad \text{such that} \quad C_2/\lambda_i T_{ref} \approx 1$$

Sensitivity matrix to the reduced parameters:

$$\mathbf{X} = \begin{bmatrix} 1 & \lambda_1^* & \lambda_1^{*2} & \dots & \dfrac{-C_2}{\lambda_1 T_{ref}} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & \lambda_N^* & \lambda_N^{*2} & \dots & \dfrac{-C_2}{\lambda_N T_{ref}} \end{bmatrix}_{N,m+2}$$



Sensitivity to the 1/T term is **very smooth**, close to linear

strong correlation between the parameters (near collinear sensitivity vectors)

Multiwavelength pyrometry. Linear least squares problem (2/5)

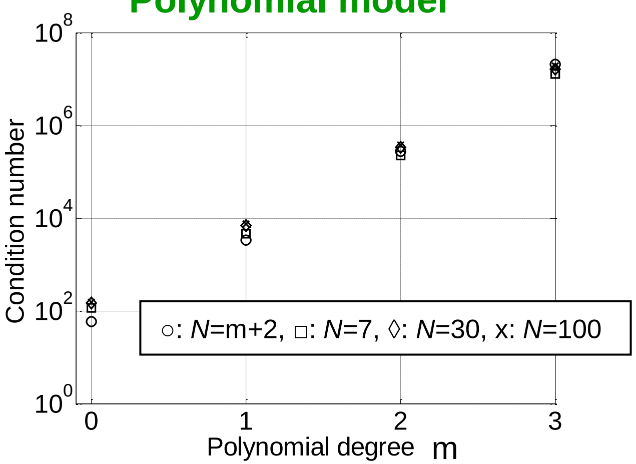
Assuming that the measurement errors are additive, uncorrelated and of uniform variance, an estimation of the parameter vector $\hat{\mathbf{P}}*$ in the least squares sense is obtained by solving the linear system :

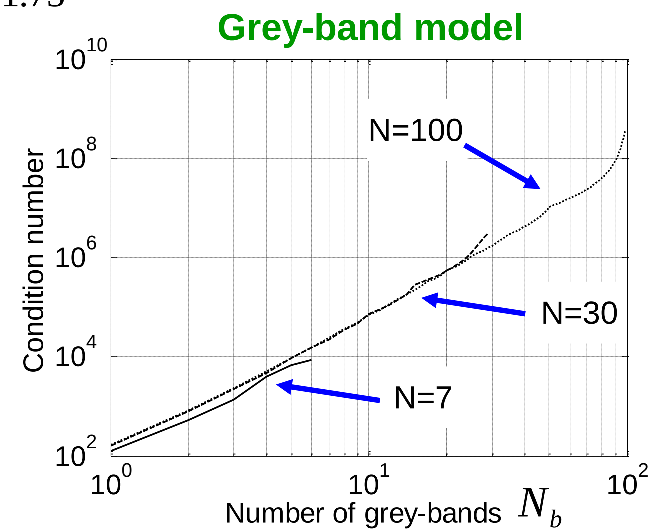
$$\left(\mathbf{X}^T\mathbf{X}\right)\hat{\mathbf{P}}* = \mathbf{X}^T\mathbf{Y}$$

Near collinear sensitivity vectors ⇒ **high condition number** of the matrix $\left(\mathbf{X}^T\mathbf{X}\right)$

The **condition number** (ratio of maximum to minimum eigenvalue) provides an upper bound of the rate at which the identified parameters will change with respect to a change of the observable (**sensitivity to measurement errors**)

$\lambda_{\max}/\lambda_{\min} = 1.75$





Huge increase with the polynomial degree or with the number of grey-bands (like N_b^3)

Bad results are expected with polynomial models of degree >1, and with grey-band model with $N_b > 7-8$

Multiwavelength pyrometry. Linear least squares problem (3/5)

Condition number : only an **upper bound** of error amplification.

The **diagonal of the covariance matrix** $\left(\mathbf{X}^T\mathbf{X}\right)^{-1}$ is of greater value for analyzing the error propagation

$$\left[\sigma_{P*}{}^2\right] = diag\left(\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\right)\sigma^2$$

assumed uniform variance of the observable

error around the mean estimator value due to **radiance error propagation to the parameters** (**does not include the bias due to the model error**, i.e. misfit between the true emissivity and the emissivity model)

Error amplification factors

K_ε

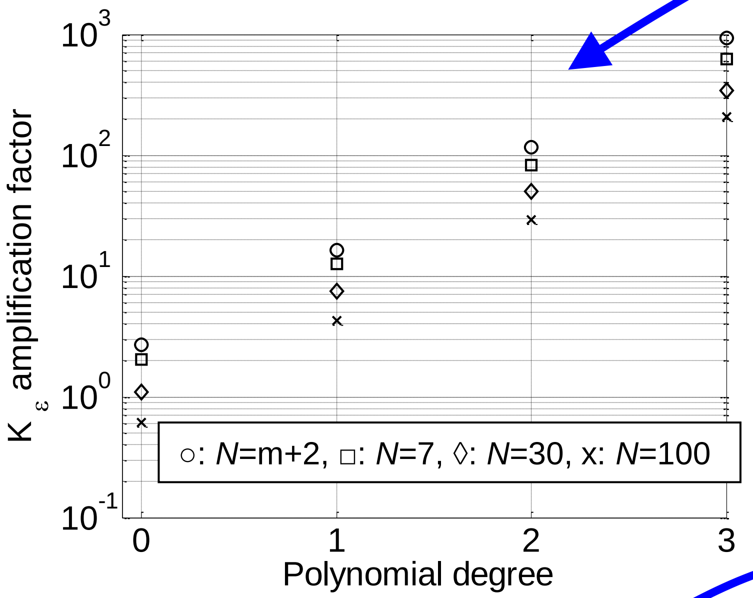
$$\frac{\sigma_\varepsilon}{\varepsilon} = K_\varepsilon \frac{\sigma_L}{L}$$

K_T

$$\frac{\sigma_T}{T} = K_T \lambda_{\min} T \frac{\sigma_L}{L}$$

Multiwavelength pyrometry. Linear least squares problem (4/5)

Polynomial model



$$\cong \frac{\exp(1.94m)}{\sqrt{N+4}}$$

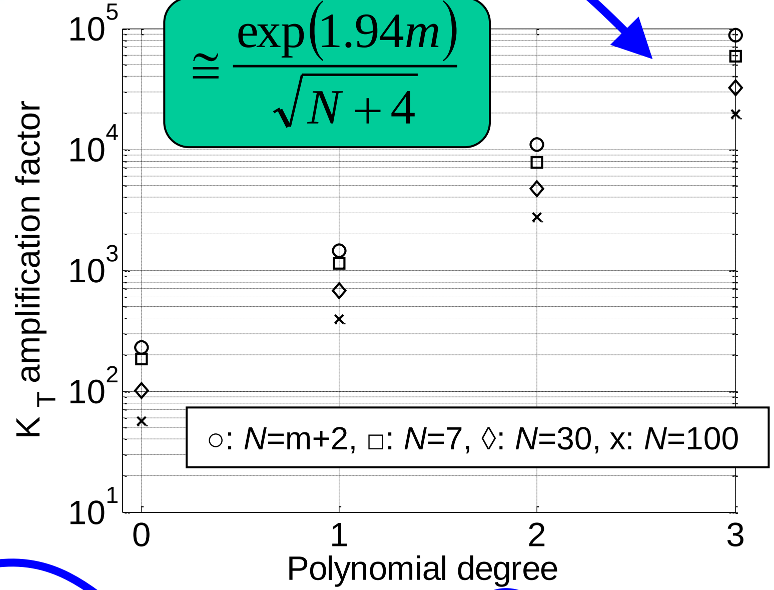
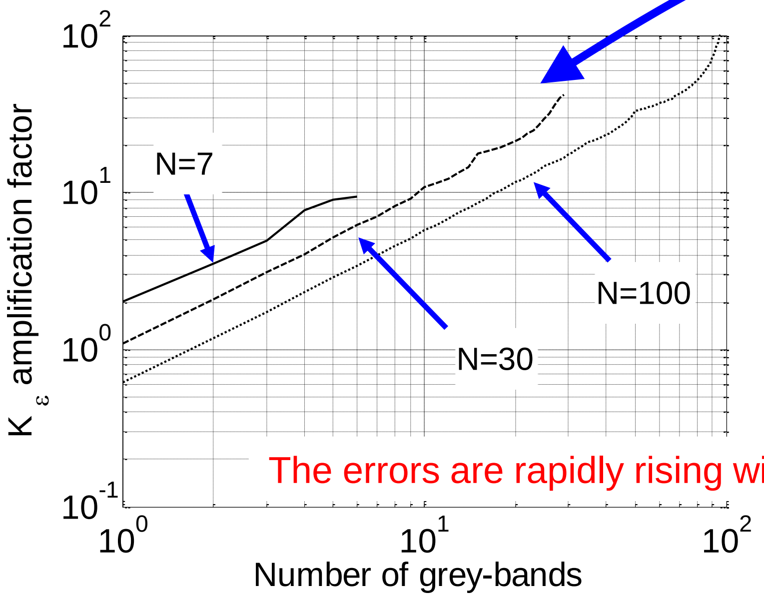


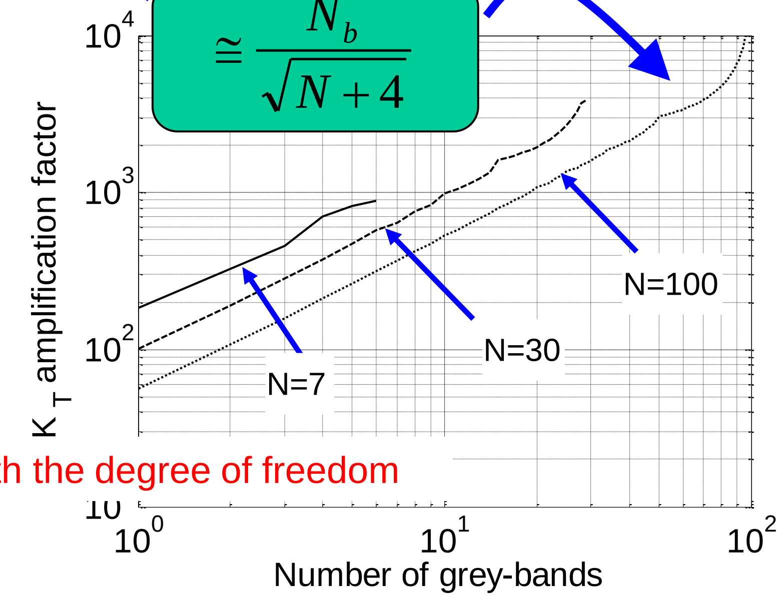
Illustration for

$\lambda_{\max}/\lambda_{\min} = 1.75$

Grey-band model



$$\cong \frac{N_b}{\sqrt{N+4}}$$



The errors are rapidly rising with the degree of freedom

Multiwavelength pyrometry. Linear least squares problem (5/5)

Application :
- target at 320K,
- 1% radiance noise
- radiometer with **seven wavelengths** between 8µm and 14µm

Polynomial model

Polynomial degree	σ_T (K)	σ_ε
0	1.5	0.02
1	9.4	0.13
2	64	0.83

Grey-band model

Number of bands	σ_T (K)	σ_ε
1	1.5	0.020
2	2.6	0.035
3	3.7	0.049
4	5.7	0.076
5	6.7	0.090
6	7.2	0.094

The mentioned standard errors only reflect what happens when noise corrupts the radiance emitted by a surface **which otherwise perfectly follows the chosen model (polynomial or staircase model)**

Multiwavelength pyrometry. Another look to the ETS problem (1/3)

The problem to solve is to find the temperature value T and the emissivity spectrum $\varepsilon(\lambda_i) \quad i=1,N$ such that (no noise at this stage):

$$L(\lambda_i,T)=\varepsilon(\lambda_i)B(\lambda_i,T) \quad i=1,N$$

There is an **infinity** of solutions. To any temperature value $\hat{T}$ one can associate an emissivity spectrum $\hat{\varepsilon}(\lambda_i,\hat{T})$ such that $\hat{T}$ and $\hat{\varepsilon}(\lambda_i,\hat{T})$ are **perfect solutions** :

$$L(\lambda_i,T)=\hat{\varepsilon}(\lambda_i,\hat{T})B(\lambda_i,\hat{T}) \quad i=1,N$$

$\hat{\varepsilon}(\lambda_i,\hat{T})$ is simply:

“true” emissivity spectrum

“true” temperature

$$\hat{\varepsilon}(\lambda_i,\hat{T})=\varepsilon(\lambda_i)\frac{B(\lambda_i,T)}{B(\lambda_i,\hat{T})} \quad i=1,N$$

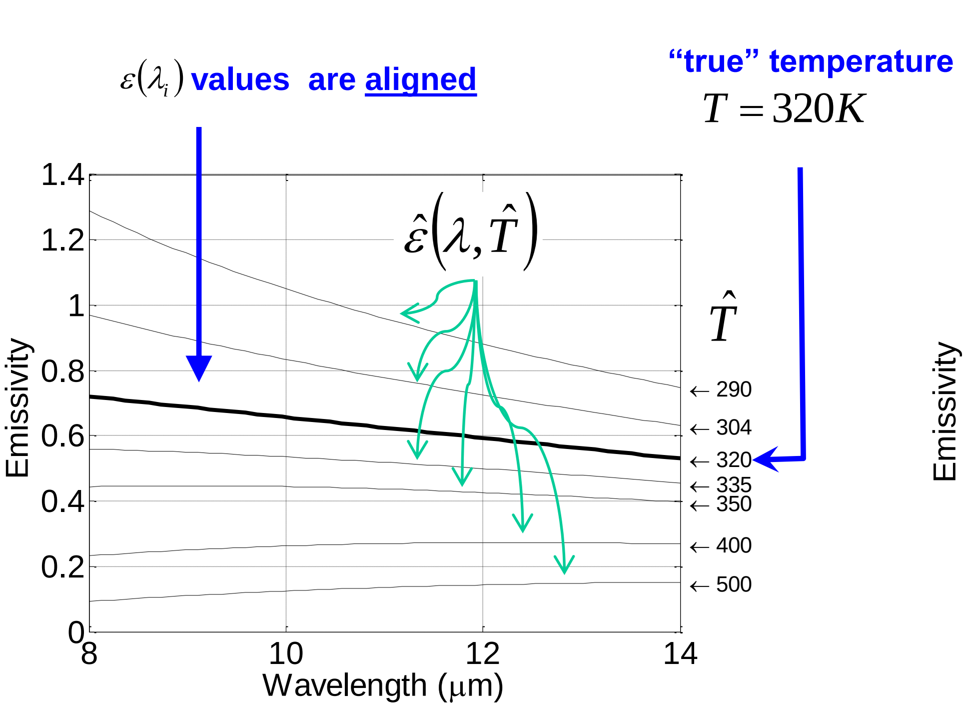
“virtual” emissivity spectrum

“virtual” temperature

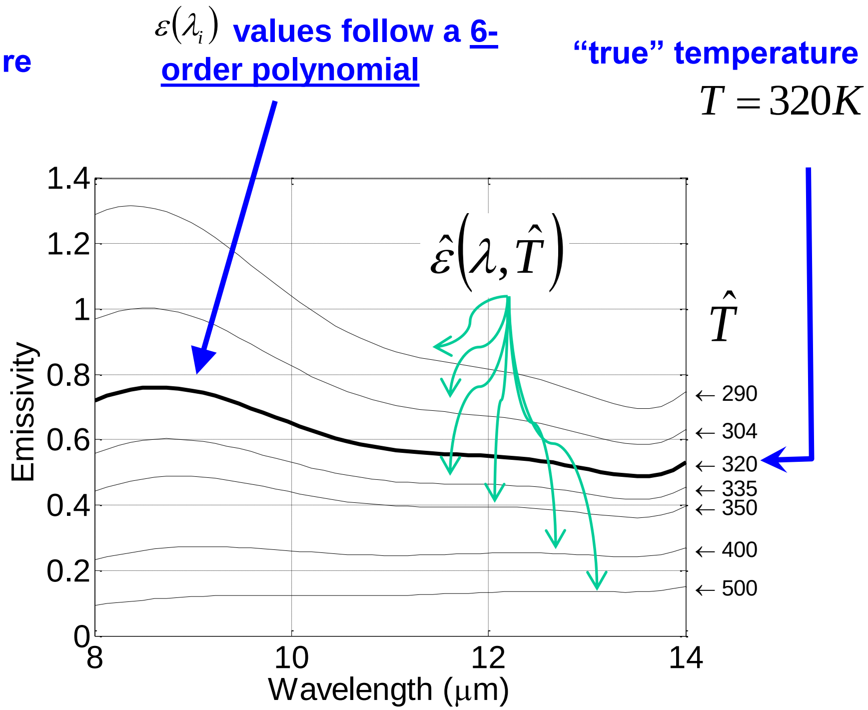
Multiwavelength pyrometry. Another look to the ETS problem (2/3)

$$\hat{\varepsilon}\left(\lambda_i,\hat{T}\right)=\varepsilon\left(\lambda_i\right)\frac{B\left(\lambda_i,T\right)}{B\left(\lambda_i,\hat{T}\right)} \qquad i=1,N$$

Real spectrum is **regular**



Real spectrum is **irregular**



Let us now consider a **1-degree** emissivity model.

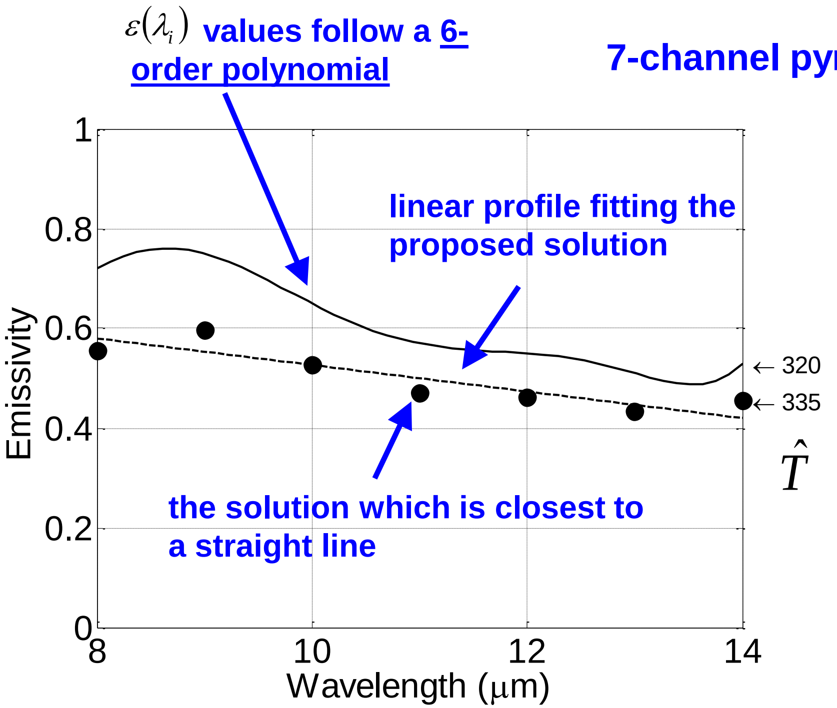
Which one, among all these candidate profiles, **is closest to a straight line ?**

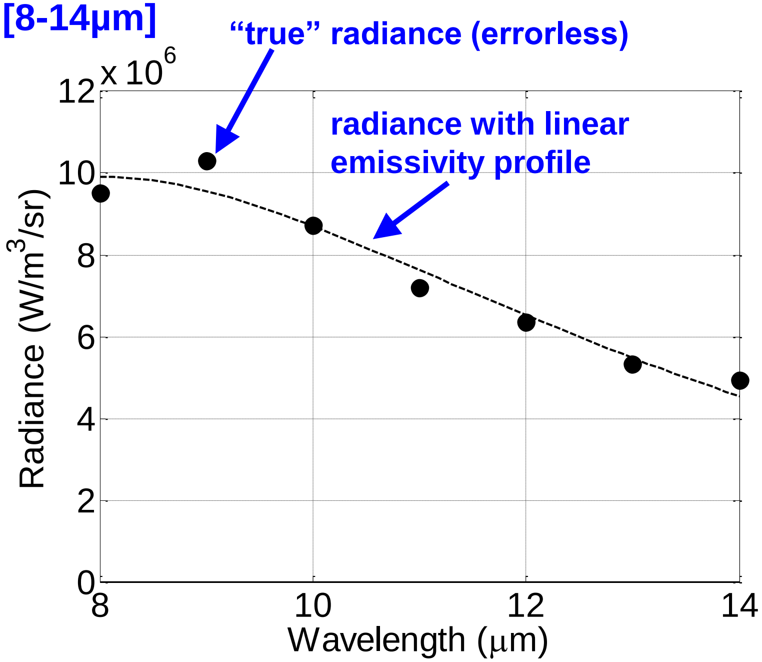
Multiwavelength pyrometry. Another look to the ETS problem (3/3)

The least squares method **selects**, among all possible solutions, **the one which conforms at best to the chosen model,** taking into account a weighting by $B(\lambda,\hat{T})$

Common (wrong) belief : « the chosen model is used **to *fit* the true emissivity profile** »

7-channel pyrometer [8-14µm]





Errorless radiance leads to a **15K temperature bias** and to **0.06 to 0.2 emissivity underestimation** (**systematic or model error**)

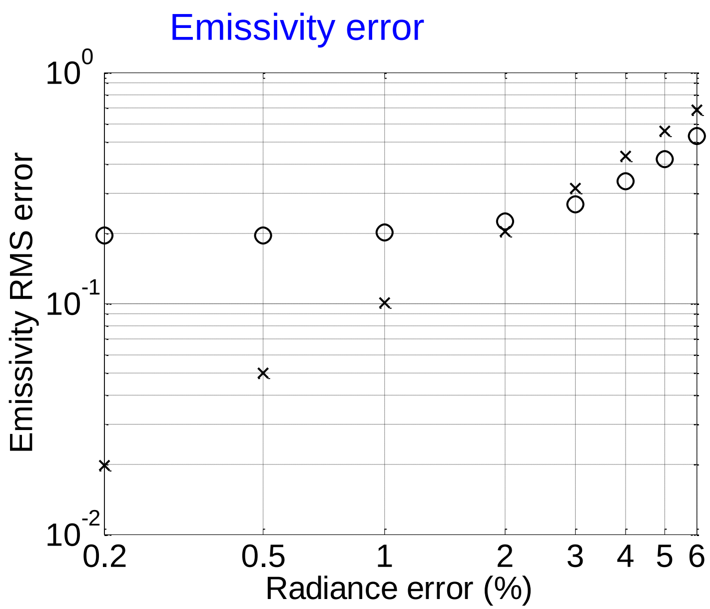
With a **2-degree** polynomial model, the results are **much worse** and even **unrealistic** :$\hat{T}$ =230K, $\hat{\varepsilon}$ >2 !

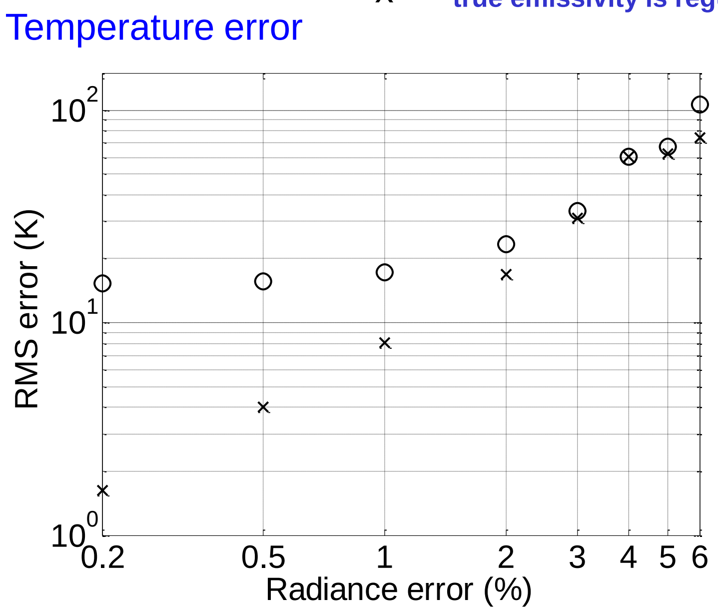
Multiwavelength pyrometry. Non-linear least squares and Monte-Carlo analysis (1/4)

- Measurements are simulated by adding **artificial gaussian noise** to the theoretical emitted radiance (std. dev.: 0.2% to 6% of the maximum radiance value)
- Statistical analysis on 200 simulated experiments
- Chosen model : **1-degree polynomial**

○ **true emissivity is irregular (6-degree polynomial)**

X **true emissivity is regular (linear)**





High systematic error when the emissivity model (**1-degree polynomial**) **does not match with the true profile** (>**15K** RMS !).
Otherwise, 0.1 emissivity error and **8K** temperature error for **1% radiance error**. Same holds when the true profile **departs by 1% from a straight line** !

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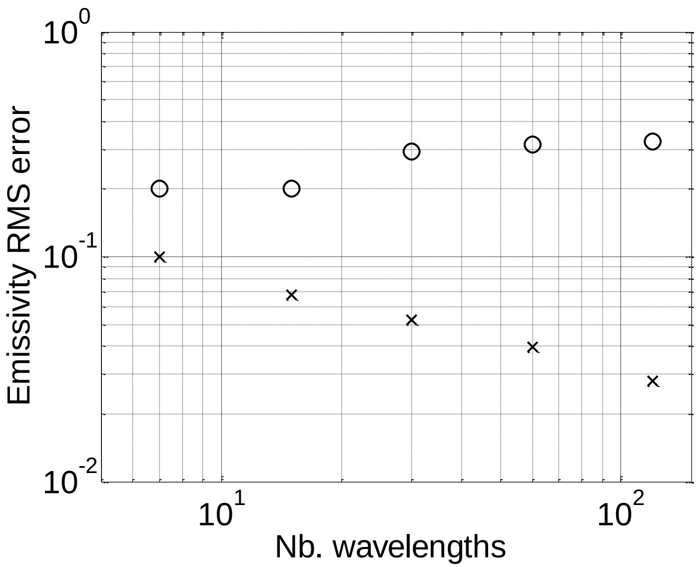
Multiwavelength pyrometry. Non-linear least squares and Monte-Carlo analysis (2/4)

- Does it help to **increase the number of spectral channels** ?

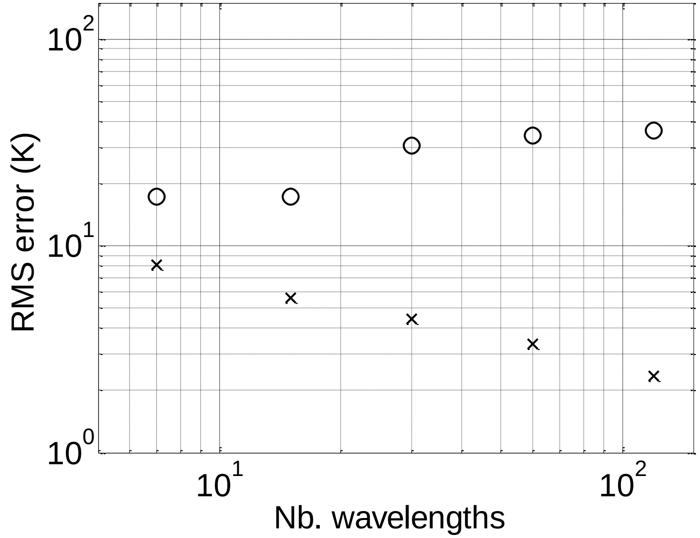
○ **true emissivity is irregular (6-degree polynomial)**

x **true emissivity is regular (linear)**

Emissivity error



Temperature error



When the emissivity model (1-degree polynomial) **perfectly matches** with the true profile we observe the classical $N^{-1/2}$ **uncertainty reduction**.
Otherwise, emissivity and temperature RMS **error remain high** (**systematic errors dominate**); they even increase with N for the presented example !
Similarly **disappointing** results with the **grey-band** model (next two slides).

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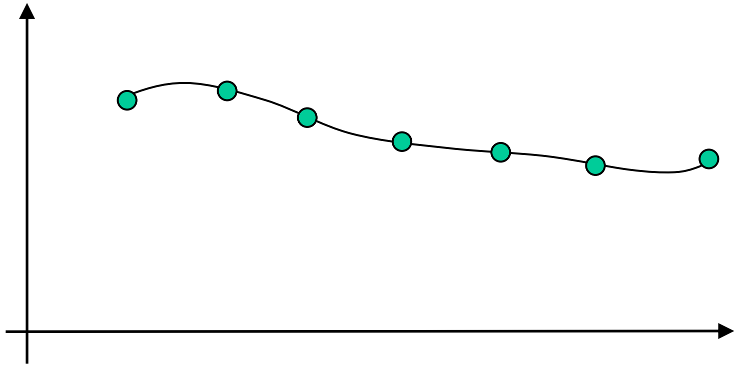
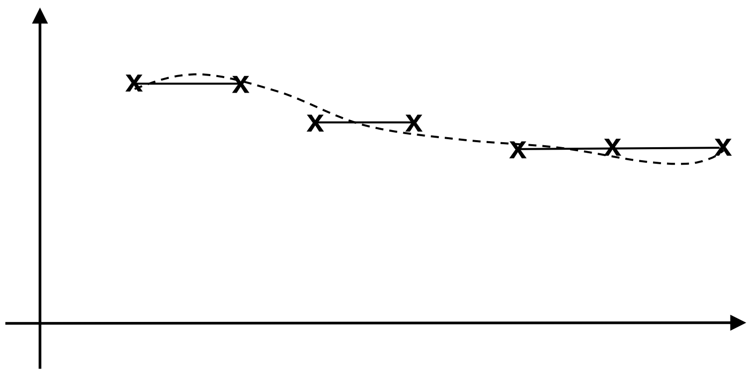
Multiwavelength pyrometry. Non-linear least squares and Monte-Carlo analysis (3/4)

- Same as before with now the **staircase model**.

Two options : the staircase model is able to fit the real emissivity profile or not



true emissivity is a 6-order polynomial sampled at N points → **the staircase model cannot match with the true profile**



x “true” emissivity is obtained after averaging a 6-order polynomial in each greyband (here N_b=3 greybands) → **the staircase model can match with this (synthetic) “true” profile**

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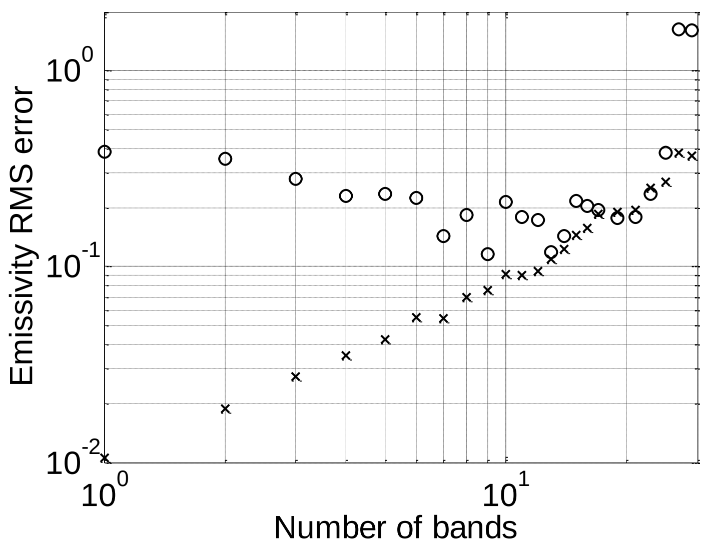
Multiwavelength pyrometry. Non-linear least squares and Monte-Carlo analysis (4/4)

- Same as before with now the **staircase model**.

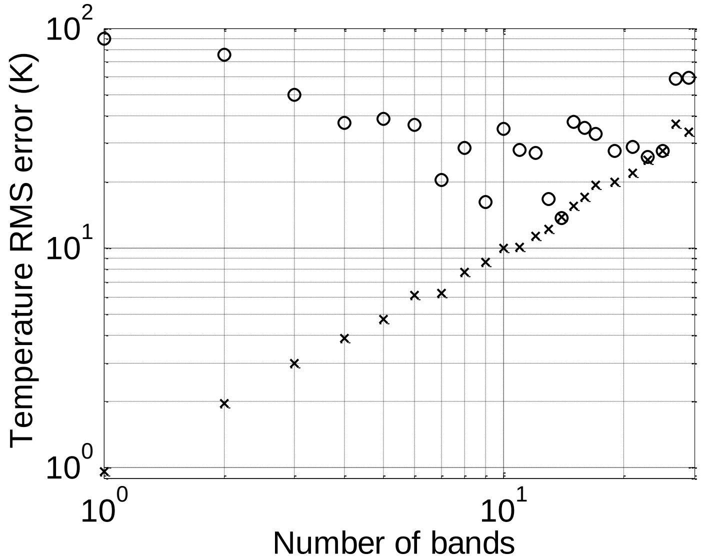
○	true emissivity is a 6-order polynomial	➡	model cannot match with the true profile
x	true emissivity is a 6-order polynomial averaged in each grayband	➡	model can match with the "true" profile

Example with N=30 channels and 1% radiance noise

Emissivity error



Temperature error



RMS errors rise proportionnaly to N_b when the model can match with the "true" profile.

Otherwise, high systematic errors. Unpredictably high variations with N_b . The least bad results are observed for intermediate values of N_b

No better results than with a linear model.

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Conclusion on LSMWP with low-order emissivity models

- Reasonable RMS values can be obtained only when the implemented emissivity model **perfectly matches** the real emissivity spectrum
- Otherwise, **important systematic errors are encountered**
- Question : when can we guaranty that **a specific emissivity model** and the **real** emissivity spectrum **perfectly match** ?
- **LSMWP** focuses on **profile shape** instead of **magnitude**

↳ One should add a penalization based on the emissivity **level** (mean or local) in order to force the solution to remain close to a predetermined **level** (*a priori* information)

↳ When using **only** the **emitted** spectral radiance, there is no valuable reason for implementing **MWP** instead of the simpler **one-color or bispectral pyrometry**

Conclusion on LSMWP with low-order emissivity models

- The **implicit weakness** of LSMWP:

$$\varepsilon_i \Leftrightarrow \varepsilon_i^{\text{model}} = \sum_{j=0}^{m} a_j \lambda_i^{\,j}$$

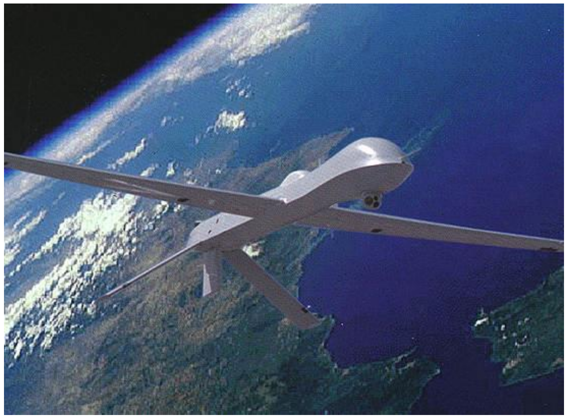
$$\chi^2 = \sum_{i=1}^{N} \sigma_i^{-2} \left(L_i - \varepsilon_i B(\lambda_i, T)\right)^2$$

Minimizing the cost function χ^2 simply means that one succeeds in getting $\varepsilon_i^{\text{model}} B(\lambda_i, T)$ close to the measurements L_i

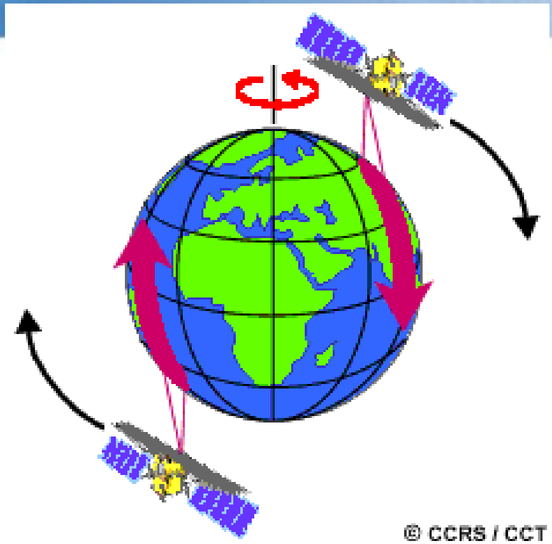
By no means $\sum \left(\varepsilon_i^{\text{real}} - \varepsilon_i^{\text{model}}\right)^2$ is expected to be « minimized » by some wizard with the help of some « hidden » process !

ETS in the field of remote sensing

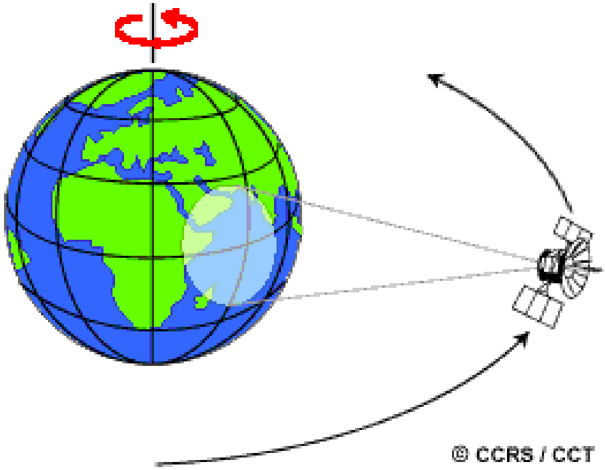
Low-altitude airborne remote sensing



High-altitude airborne remote sensing

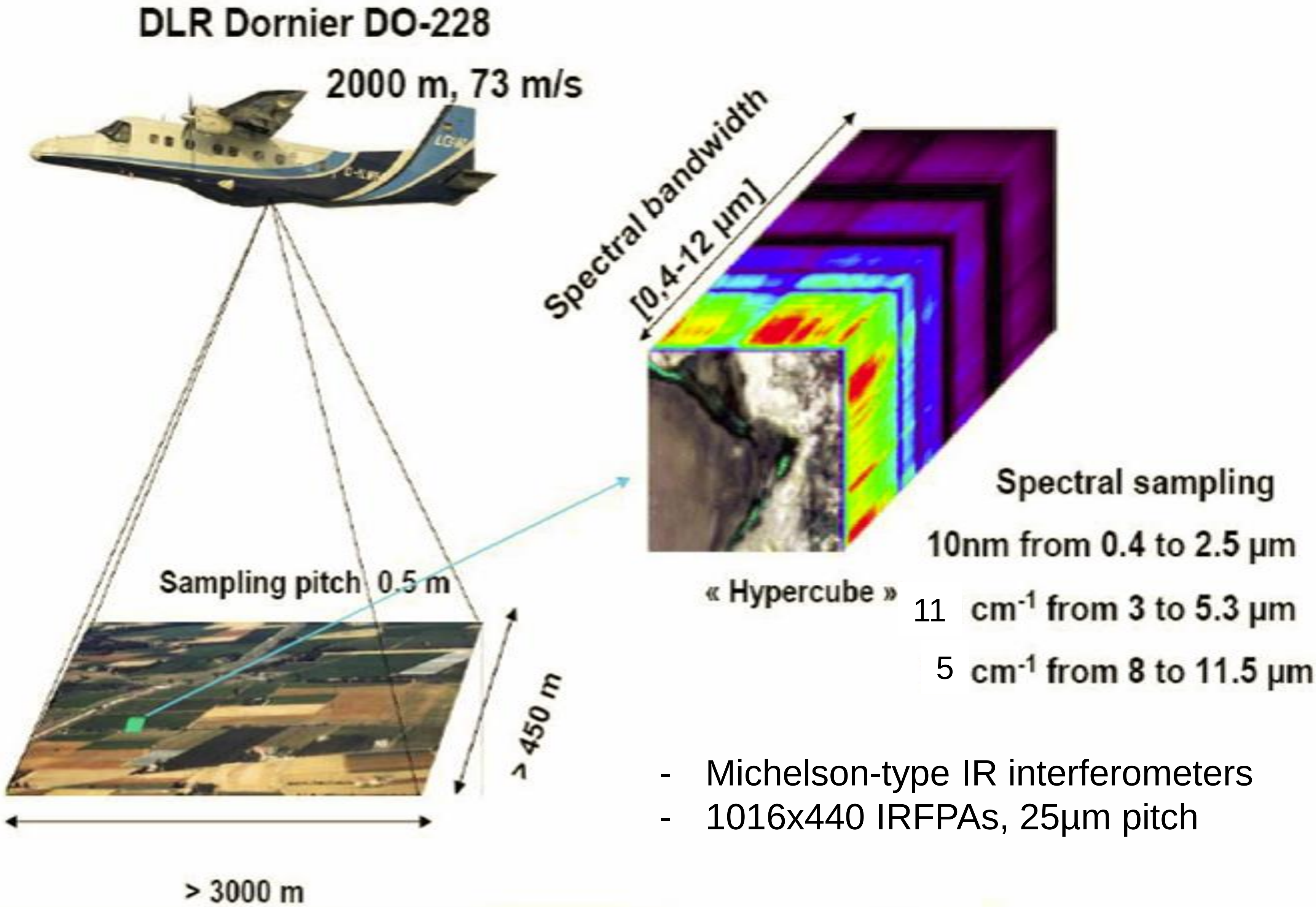


Polar-orbiting satellites (low-earth orbit)



Geostationary satellite

SYSIPHE main caracteristics



- Michelson-type IR interferometers
- 1016x440 IRFPAs, 25µm pitch

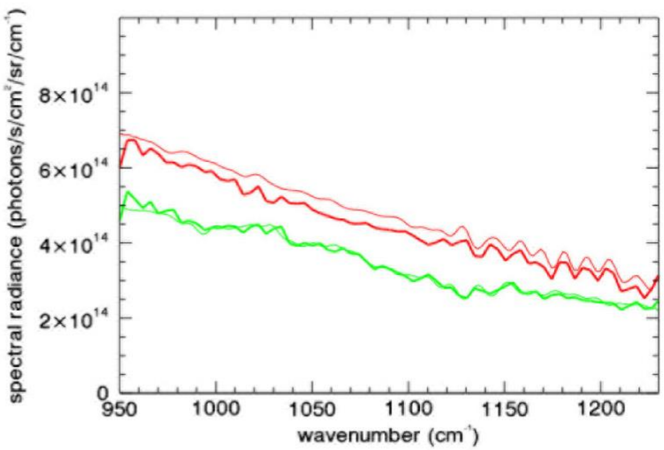
First results of the dual band MWIR+LWIR spectro-imaging system SIELETERS

Monochromatic image (λ = 4.8µm, Δλ = 0.025µm) from the MWIR hyperspectral cube.

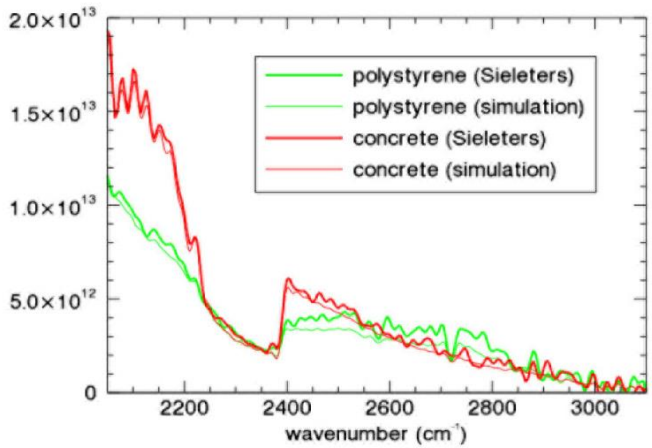
Green cross : polystyrene target
Red cross : concrete area

Coudrain, Opt. Exp. 2015

Spectra of two pixels: polystyrene target (green), concrete target (red), for the LWIR (a) and the MWIR (b).



(a)



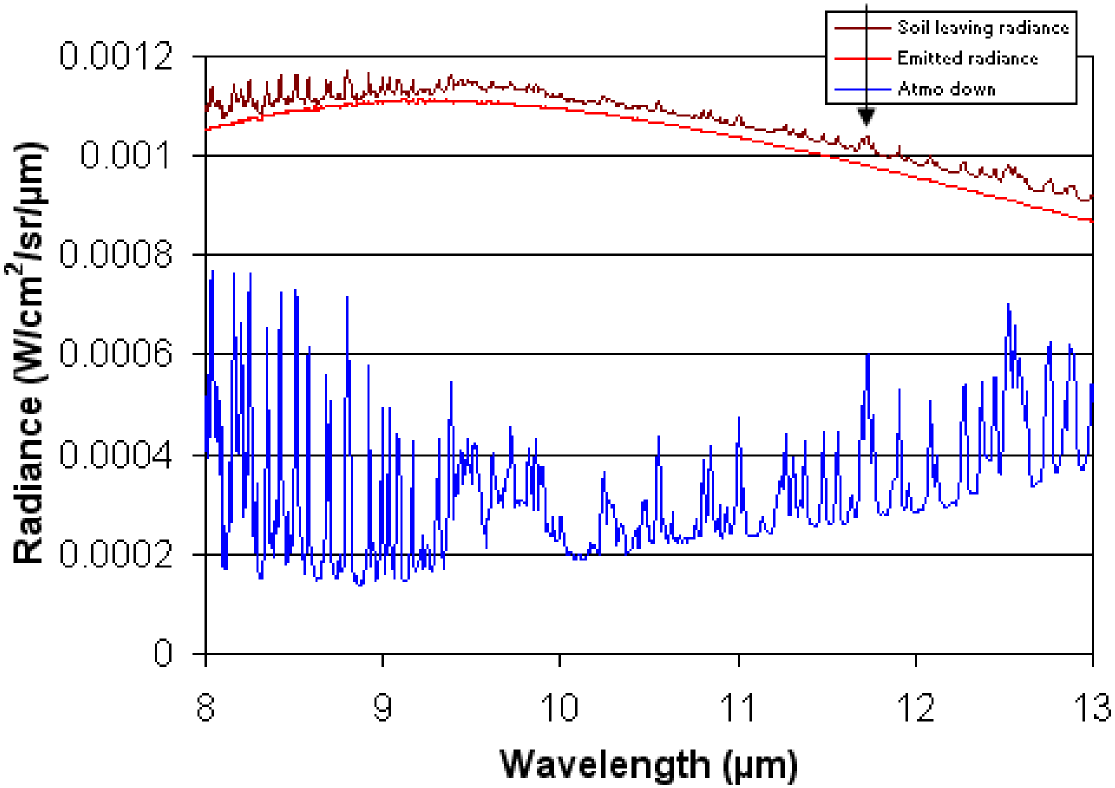
(b)

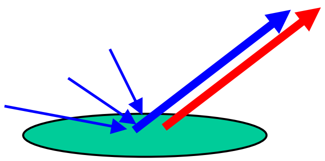
Specific features of IR remote sensing

- Measurements are highly conditioned by the **radiative properties of the atmosphere** (transmission, emission toward the earth surface and then reflection, emission along the optical path, scattering, …).
- Optical path in air from ~100 m to several km.
  - **Atmosphere compensation** is necessary
  - Atmosphere properties are considered **uniform** in images of several km^2
- Footprint is generally large: from ~10 cm for low altitude airborne sensors to ~2 km for sensors on geostationary satellites $\Rightarrow$ **aggregation** of various materials and temperatures (**desaggregation = inversion problem**)
- In [8-14µm] band, natural surfaces (soil, vegetation, water) have **high emissivity values** (> 0.9). Generally considered as **Lambertian**.

Evaluation of atmosphere contributions

- Example of a grey surface (ε=0.9) at T=313K
- Radiative transfer simulations with MODTRAN; (mid-latitude summer atmospheric model; rural aerosols)

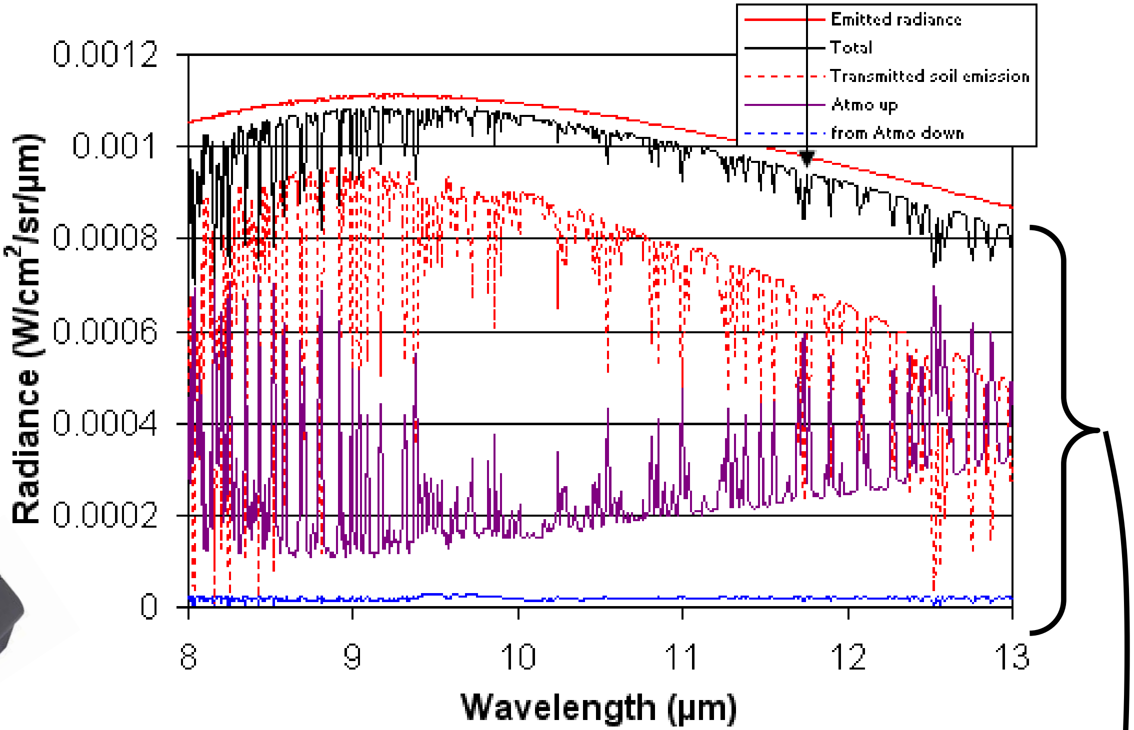




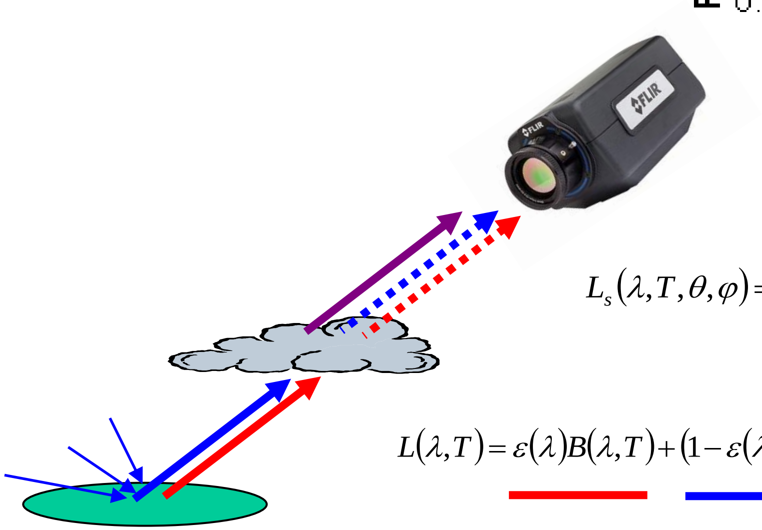
$$L(\lambda,T)=\varepsilon(\lambda)B(\lambda,T)+(1-\varepsilon(\lambda))L^{\downarrow}(\lambda)$$

Evaluation of atmosphere contributions

- Example of a grey surface (ε=0.9) at T=313K sensed by an IR instrument at 1900 m altitude.
- Radiative transfer simulations with MODTRAN; (mid-latitude summer atmospheric model; rural aerosols)



at-sensor radiances



$$L_s(\lambda,T,\theta,\varphi)=\tau(\lambda,\theta,\varphi)L(\lambda,T)+L^{\uparrow}(\lambda,\theta,\varphi)$$

$$L(\lambda,T)=\varepsilon(\lambda)B(\lambda,T)+(1-\varepsilon(\lambda))L^{\downarrow}(\lambda)$$

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Atmosphere separate compensation

Radiative transfer simulation (MODTRAN, MATISSE…) with:

- **standard atmospheric models** (temperature+humidity profiles)/climate/season/aerosols

- **radiosonde data** $\Longrightarrow$ profiles of pressure, temperature, constituents

- **IR sounding near** 4.3µm for CO_2 and between 4.8-5.5µm for H_2O + neural networks allows retrieving **mean atmosphere temperature** and **columnar water vapor** under the sensor. These values are then used to scale a set of standard atmosphere profiles used in MODTRAN and get closer to the true atmosphere profiles. Final MODTRAN computation

$$\Rightarrow \begin{cases} \tau(\lambda,\theta,\varphi) \\ L^{\uparrow}(\lambda,\theta,\varphi) \\ L^{\downarrow}(\lambda) \end{cases}$$

Emissivity-Temperature separation

- Proper **atmosphere compensation** provides **ground leaving radiance:**

$$L(\lambda,T)=\frac{L_s(\lambda,T)-\underline{L^{\uparrow}(\lambda)}}{\underline{\tau(\lambda)}}=\underline{\varepsilon}(\lambda)B(\lambda,\underline{T})+(1-\underline{\varepsilon}(\lambda))\underline{L^{\downarrow}}(\lambda)$$

Emissivity estimation $\hat{\varepsilon}(\lambda)$ from a temperature estimation $\hat{T}$ according to $\boxed{\hat{\varepsilon}(\lambda)=\frac{L(\lambda,T)-L^{\downarrow}(\lambda)}{B(\lambda,\hat{T})-L^{\downarrow}(\lambda)}}$

- **SpSm** method (**Spectral Smoothness**)
  - emissivity spectrum is far **smoother** than downwelling radiance

- **Multi-temperature** inversion
  - performing measurements at least at two **different** temperature levels

$\left\{\begin{array}{l}\text{one more unknown}\\ \text{N more data}\end{array}\right.$ ⇨ **is the ill-conditionning solved ???**

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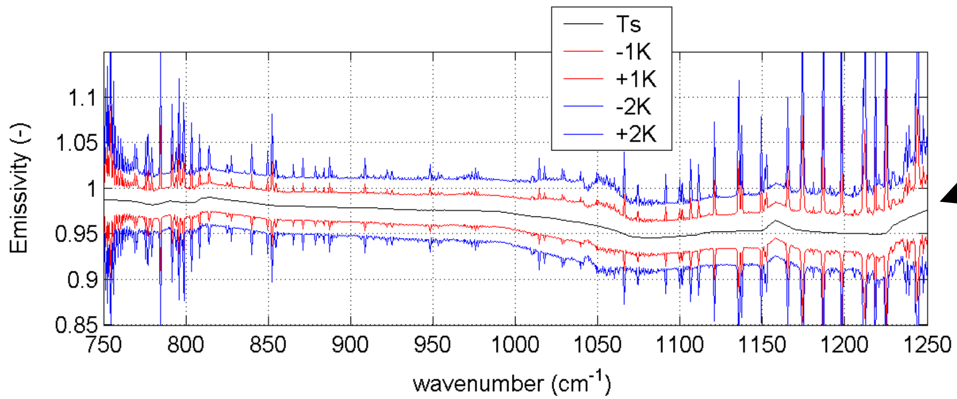
Spectral Smoothness method (SpSm)

- When the temperature estimation $\hat{T}$ **is in error**, the profile will contain **detailed spectral features** originating from $L(\lambda,T)$ and $L^{\downarrow}(\lambda)$ (gas absorption bands)

$$\hat{\varepsilon}(\lambda)=\frac{L(\lambda,T)-L^{\downarrow}(\lambda)}{B(\lambda,\hat{T})-L^{\downarrow}(\lambda)}$$

- Adjust $\hat{T}$ until $\hat{\varepsilon}(\lambda)$ is **deprived of these artifacts** → **"smooth" emissivity spectrum**



Knuteson, 2006

- **Smoothness criteria** : minimization of std. dev. between $\hat{\varepsilon}(\lambda)$ and its local mean, correlation product between $\hat{\varepsilon}(\lambda)$ and $L^{\downarrow}(\lambda)$, …
- SpSm requires the **atmospheric compensation to be very precise**
- SpSm requires **high spectral resolution** (< 10 cm^{-1}) in order to capture sufficient details of the atmosphere spectral features. Restricted to **hyperspectral data**. Spectral calibration errors are highly detrimental
- Radiance error of 0.5% → 1.6K RMS and 0.8K bias for temperature and 0.023 RMS and 0.027 bias for emissivity

Multi-temperature method : a pitfall ? (1/3)

one additional unknown parameter but **N more data !**

Does it solve the ill-conditionning ???

- N_T temperature levels
- N channels

- $N+N_T$ unknowns
- $N\mathrm{x}N_T$ equations

Solvable (**in principle**) as soon as $N \geq 2$

However, when using **Wien 's** approximation, it can be shown that, when there is **no reflection** contribution, the problem **remains ill-conditionned** !

- With errorless radiance, there is an infinite number of solutions defined by:

$$\begin{cases} \dfrac{1}{\hat{T}_t} = \dfrac{1}{T_t} + cst \\ \hat{\varepsilon}(\lambda) = \varepsilon(\lambda)\exp\left(\dfrac{C_2}{\lambda} cst\right) \end{cases}$$

- In case of two temperature measurement, the sensivity matrix is:

$$\mathbf{X} = \begin{bmatrix} & \dfrac{-C_2}{\lambda_1 T_{ref}} & 0 \\ \mathbf{I}_N & \cdots & \cdots \\ & \dfrac{-C_2}{\lambda_N T_{ref}} & 0 \\ & 0 & \dfrac{-C_2}{\lambda_1 T_{ref}} \\ \mathbf{I}_N & \cdots & \cdots \\ & 0 & \dfrac{-C_2}{\lambda_N T_{ref}} \end{bmatrix}_{2N,N+2}$$

The sensitivities are correlated since

$$\det\left(\mathbf{X}^T\mathbf{X}\right) = 0 \quad !!$$

However, degeneracy could be alleviated **thanks to the presence of reflections**

Inversion robustness depends (again) on the **spectral richness** of the reflected radiance

Multi-temperature method (2/3)

- The problem remains badly conditioned when using Planck's law
- Degeneracy is alleviated thanks to the presence of reflections
- Inversion robustness depends on the spectral richness of the reflections

Case of two temperatures.

Nonlinear least-squares approach for identifying the N emissivities and the two temperatures

$$[\varepsilon_i, T_1, T_2]^T = \underset{\varepsilon_i, T_1, T_2}{\arg Min} \sum_{i=1}^{N} \begin{array}{l} \left(L(\lambda_i, T_1) - \left(\varepsilon_i B(\lambda_i, T_1) + (1-\varepsilon_i) L^{\downarrow}(\lambda_i)\right)\right)^2 + \\ \left(L(\lambda_i, T_2) - \left(\varepsilon_i B(\lambda_i, T_2) + (1-\varepsilon_i) L^{\downarrow}(\lambda_i)\right)\right)^2 \end{array}$$

Illustration for the case of a greybody (ε=0.9) at T_1 =320K.

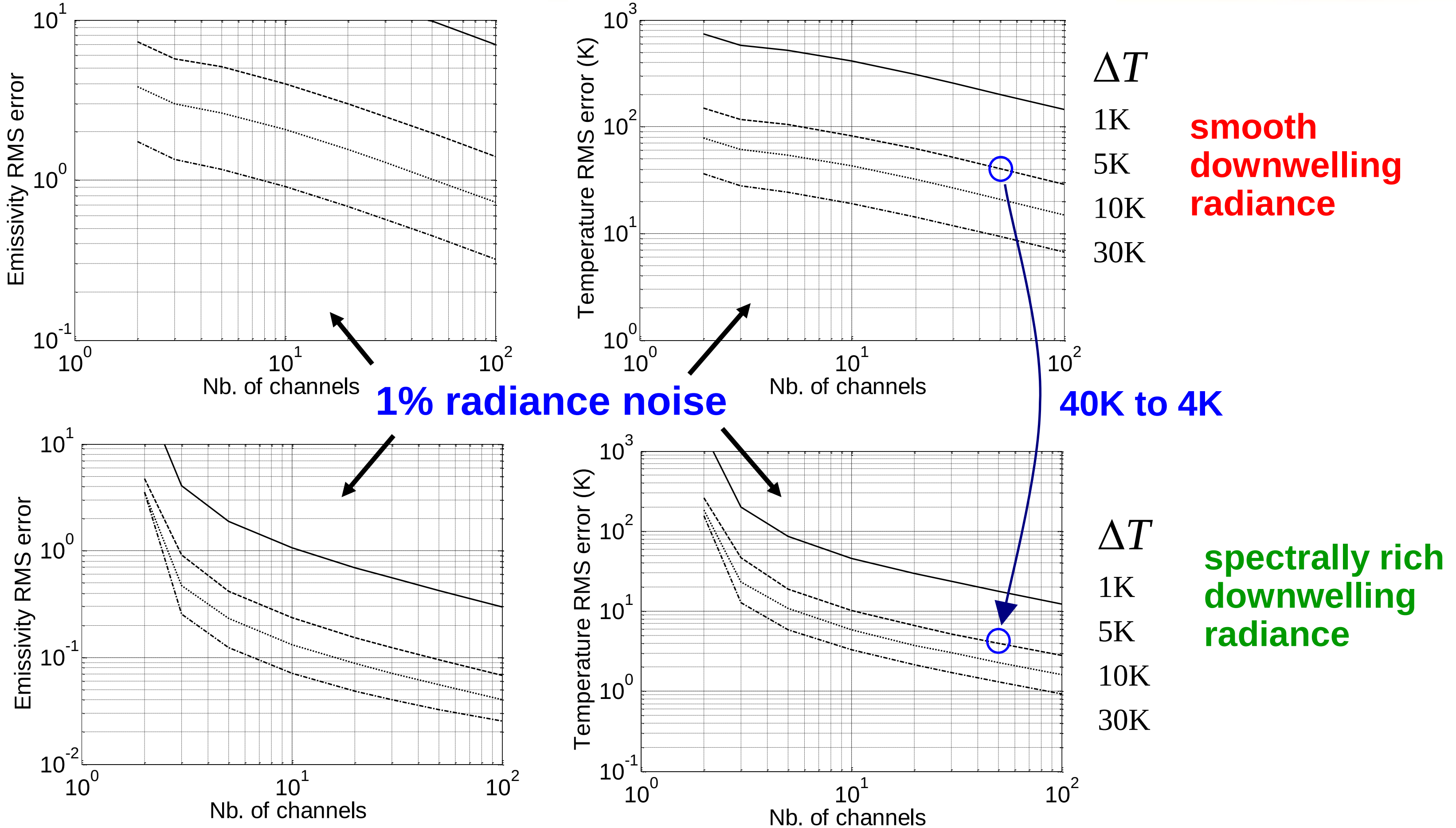
Second temperature is 1K, 5K, 10K or 30K higher.

Downwelling radiance is either:

- blackbody radiance at 300K
- same by weighting with a uniform random distribution (simulation of the presence of detailed spectral features)

Standard errors of identified parameters obtained from covariance matrix (local linearization)

Multi-temperature method (3/3)



Better results are obtained by increasing the number of channels and the temperature difference

High constraints to get a temperature RMS error lower than 1K !

Constraints on images co-registration, on emissivity stability.

Pyrometry : a Bayesian perspective (1/3)

- Unknown parameters : $\varepsilon(\lambda_i), T \qquad i = 1, N$
- Parameters are uncertain; the information available before the observation is represented by their prior distributions : $p(\varepsilon_i), p(T)$
- Experimental data are radiance observations:

$$y_i = L(\varepsilon_i, T) + e_i$$

- The conditional propability for measuring $y_i \quad i = 1, N$ given the parameters is the likelihood:

$$p(y|\varepsilon, T) = p(y - L(\varepsilon, T))$$

Classical inference is looking for the parameters values that maximise the likelihood

When the noise is Gaussian with covariance matrix Ψ *:*

$$p(y|\varepsilon, T) \propto \exp\left[-\frac{1}{2}(y - L(\varepsilon, T))^T \Psi^{-1}(y - L(\varepsilon, T))\right]$$

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Pyrometry : a Bayesian perspective (2/3)

- Bayesian inference about the parameters is based on the posterior probability density as conditioned by the data (Bayes theorem):

$$p(\varepsilon, T|y) = \frac{p(y|\varepsilon, T)p(\varepsilon)p(T)}{p(y)}$$

The denominator is considered as a normalizing constant, the analysis is performed about:

$$p(\varepsilon, T|y) \propto p(y|\varepsilon, T)p(\varepsilon)p(T)$$

$\Rightarrow$ *mode(s), mean, confidence intervals*

- When one is essentially interested in evaluating temperature, emissivity can be considered as a nuisance variable $\Rightarrow$ marginalization

$$p(T|y) \propto \int p(\varepsilon, T|y) d\varepsilon_1 d\varepsilon_2 \ldots d\varepsilon_N$$

- Priors :

Non-informative:

- $p(T) = dT/T$ (T acts as a scale)
- $p(\varepsilon_i) = d\varepsilon_i I_{[0,1]}$ (location parameter)

Specific:

- $T \sim U_{[T_{\min}, T_{\max}]}$
- $\varepsilon_i \sim U_{[\varepsilon_{\min}, \varepsilon_{\max}]}$
- Joint propability $p(\varepsilon)$ from analysis of databases, eventually coupled with other information (ex: knowledge on ageing, corrosion, expected vegetation species…)

Pyrometry : a Bayesian perspective (3/3)

- Simple example : 1-color pyrometry

$$e_1 \sim N(0,\sigma^2)$$
$$\varepsilon_1 \sim U(\varepsilon_{\min},\varepsilon_{\max})$$

$$p(\varepsilon_1,T|y_1) \propto p(y_1|\varepsilon_1,T)\,p(\varepsilon_1)\,p(T)$$

$$p(\varepsilon_1,T|y_1) \propto \exp\left[-\frac{1}{2\sigma^2}\left(y_1-\varepsilon_1 B(\lambda_1,T)\right)^2\right] d\varepsilon_1 \frac{dT}{T}$$

- Marginalization to get rid of the nuisance parameter (emissivity):

$$p(T|y_1) \propto \frac{1}{TB(\lambda_1,T)}\left[erf\left(\frac{\varepsilon_{\max}B(\lambda_1,T)-y_1}{\sqrt{2}\sigma}\right)-erf\left(\frac{\varepsilon_{\min}B(\lambda_1,T)-y_1}{\sqrt{2}\sigma}\right)\right]$$

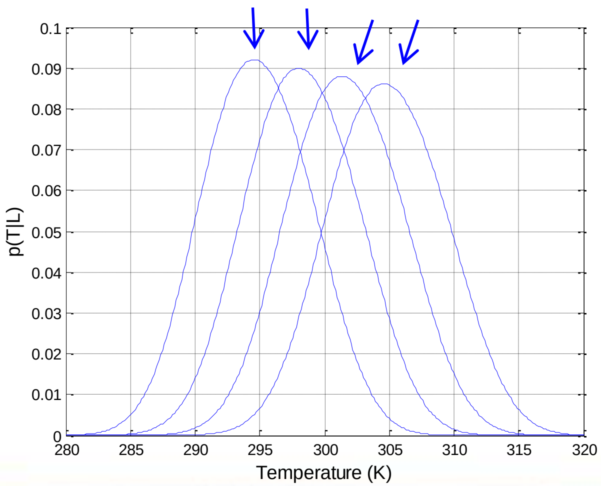
- Example : T=300K

$\lambda = 10\mu m \quad \sigma/L = 5\%$

$\varepsilon_{\min} = 0.85 \quad \varepsilon_{\max} = 1$

Posterior distribution for temperature in case of **four** different radiance values

The « effective » emissivity as considered for computing the input raw radiance was:
0.85, 0.9, 0.95, 1



- The modes are within +/-5K of the true temperature
- Possibility to evaluate the mean, the std dev., the quartiles…
- For more complex situations (higher number of parameters, more wavelengths) one possibility is to apply **Markov Chain Monte Carlo methods**

Conclusion

- Radiative temperature measurement
  - advantage : **non-contact**
  - disadvantage : **under-determined** inverse problem due to unknown emissivity
- « Mirage » or « lure » of **multiwavelength pyrometry**
  - only **very low order** emissivity models have a chance to provide useful results
  - **no significant benefit** with respect to single or two color pyrometry

  except in case of well-characterized target material
- « Mirage » or « lure » of the **multi-temperature method**
  - ineffective except in case of reflections from **spectrally rich** environment
  - additional constraints
- IR remote sensing takes profit from the **high emissivity** of natural surfaces and from their **spectral smoothness** with respect to downwelling radiance
- Bayesian methods
  - High flexibility for integration of prior information on emissivity (i.e. expected materials, expected surface state, …)

Further reading:

Krapez, J. C. (2011). *Radiative measurements of temperature*. In: *Thermal measurements and inverse techniques.* CRC Press, Taylor & Francis Group, Chap. 6. p. 185-230.

https://www.crcpress.com/Thermal-Measurements-and-Inverse-Techniques/Orlande-Fudym-Maillet-Cotta/p/book/9781439845554

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